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Two-Station Single Track Scheduling Problem ⋆

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Abstract: Single track segments are common in various railway networks, in particular in various supply chains. For such a segment, connecting two stations, the trains form two groups, depending on what station is the initial station for the journey between these two stations. Within a group the trains differ by their cost functions. It is assumed that the single track is sufficiently long so several trains can travel in the same direction simultaneously. The paper presents polynomial-time algorithms for different versions of this two-station train scheduling problem with a single railway track. The considered models differ from each other by their objective functions.

Keywords: Operations Research, Transportation science, Supply chains and networks

1. THE CONSIDERED PROBLEM

The connection of two stations by a single railway track is common in various supply chains such as supply chains of minerals, for instance coal, and supply chains of agricultural products, for example sugar cane, as well as in the manufacturing environment. Due to their practical significance and challenging mathematical nature, the scheduling problems, where trains are using a single railway track, remained the subject of intensive research since the pioneer publications Frank (1966) and Szpigel (1973).

We investigate the problem where trains travel between two stations, Station 1 and Station 2, connected by a single track. Each train travels either from Station 1 to Station 2 (the set of all such trains will be denoted by N_1), or from Station 2 to Station 1 (the set of all such trains will be denoted by N_2). Let $N = N_1 \cup N_2$.

It is convenient to number all the trains departing from the same station, i.e. it is convenient to consider $N_1 = \{1, ..., n\}$ and $N_2 = \{1', ..., n'\}$. Observe, that $1', ..., n'$ are ordinary numbers assigned to the trains departing from Station 2. For example, $n' - 1'$ gives the number associated with one of the trains departing from Station 2, whereas $n + n'$ is the total number of trains.

All trains travel with the same constant speed. The journey between Station 1 and Station 2 takes $p > 0$ units of time. The transportation starts at time $t = 0$.

At any point in time, the distance between any two trains, simultaneously moving in the same direction, must be not less than the certain minimal safe distance. In order to ensure this restriction, the difference between any two departure times from the same station can not be less than some given β. In what follows, it is assumed that $\beta < p$. In other words, it is assumed that Station 1 and Station 2 are sufficiently far apart and several trains can travel simultaneously in the same direction. Observe that the difference between any two departure times from different stations can not be less than p.

A schedule σ specifies for each train $j \in N$ the departure time $S_j(\sigma)$ and the arrival time $C_j(\sigma)$, where

$$C_j(\sigma) = S_j(\sigma) + p.$$

Each train $j \in N$ has the associated nondecreasing cost function $\varphi_j(\cdot)$. The goal is to find a schedule which minimises the objective function

$$\varphi_1(C_1(\sigma)) \odot \ldots \odot \varphi_n(C_n(\sigma)) \odot \varphi_{1'}(C_{1'}(\sigma)) \odot \ldots \odot \varphi_{n'}(C_{n'}(\sigma)),$$

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where $\odot$ is some commutative and associative operation such that for any numbers a_1, a_2, b_1, b_2, satisfying $a_1 \le a_2$ and $b_1 \le b_2$,

$$a_1 \odot b_1 \le a_2 \odot b_2. \tag{1}$$

For example, the operation $\odot$ can be addition. In this case for any $i \in N$ and $j \in N$

$$\varphi_i(C_i(\sigma)) \odot \varphi_j(C_j(\sigma)) = \varphi_i(C_i(\sigma)) + \varphi_j(C_j(\sigma)).$$

Another commonly used operation is maximum. In this case

$$\varphi_i(C_i(\sigma)) \odot \varphi_j(C_j(\sigma)) = \max\{\varphi_i(C_i(\sigma)), \varphi_j(C_j(\sigma))\}.$$

In particular, the objective function, where the operation is maximum and each $\varphi_i(x) = x$, is referred to in scheduling as the makespan and is denoted by C_{max}, i.e.

$$C_{max}(\sigma) = \max_{i \in N} C_i(\sigma).$$

Following the notation of the article Gafarov et al. (2015), the problem, considered in our paper, can be denoted by $STR2 \,|\,| \odot \varphi_j$, where STR stands for "single track railway" and 2 indicates that two stations are considered.

Since the objective function is nondecreasing, in what follows, without loss of generality, it is assumed that each schedule σ should possess the following property: for any point in time t such that

$$0 \le t \le C_{max}(\sigma)$$

there exists at least one train $i \in N$ satisfying the condition

$$S_i(\sigma) \le t \le C_i(\sigma).$$

The rest of the paper is organised as follows. Section 2 discusses the contribution of the results, presented in this paper, to the existing knowledge on train scheduling and provides a brief literature survey. Section 3 shows that if the order in which the trains, constituting the set N_1, depart from Station 1 and the order in which the trains, comprising the set N_2, depart from Station 2 are known, then, for any operation $\odot$, an optimal schedule can be obtained in polynomial time by means of dynamic programming. This section also discusses some important cases of cost functions and operation $\odot$, where the computational complexity of the general optimisation procedure can be significantly reduced. Section 4 focuses on the maximum cost objective function frequently used in practice and theory.

2. EXISTING LITERATURE

Surveys of the railway planning processes, models and methods can be found in Lusby et al. (2011), Oliveira (2001) and Harrod (2012). The existing literature covers a broad variety of models, assumptions, and practical situations. Many publications stress the importance of the single track train scheduling from the practical as well as theoretical viewpoints. Indeed, the single track scheduling problems have attracted the considerable attention, and starting from Szpigel (1973), there exists a sturdy stream of publications.

In train scheduling, one of the parameters that vary from model to model is the speed of trains. For example, similar to our paper, Harbering et al. (2015) considers the situation where all trains travel with the same constant speed, whereas in Kraay et al. (1991) the speed can vary.

The optimisation methods also vary from publication to publication. Thus, similar to our paper, Harbering et al. (2015) uses dynamic programming. Other optimisation methods include integer programming Brannlund et al. (1998), branch and bound method Higgins et al. (1996), and heuristics Carey and Lockwood (1995) and Mu and Dessouky (2011). The computational complexity results can be found in Disser et al. (2015).

The existing publications on single track train scheduling consider a wide range of objective functions, including makespan, total tardiness, maximum lateness, total completion time, etc. Our paper demonstrates that many commonly used objective functions share certain properties that allow the development of uniform optimisation procedures.

The recent publication Gafarov et al. (2015) is closely related to our paper. Similar to our paper, Gafarov et al. (2015) is concerned with the two-station case and various objective functions (although not in such general form as in our paper). In contrast to our paper that considers the safe distance between the trains specified by β, Gafarov et al. (2015) assumes that the track is partitioned into several segments and trains are not allowed to travel simultaneously along each of these segments. These two assumptions are in some sense equivalent and, as far as Gafarov et al. (2015) is concerned, the main contribution of our paper is the development of more efficient optimisation algorithms for several objective functions, considered in Gafarov et al. (2015), as well as the presented optimisation procedure for the maximum cost problem with arbitrary nondecreasing cost functions.

Harbering et al. (2015) analyses the situation with more than two stations and presents a dynamic programming based pseudo-polynomial algorithm for the makespan minimisation problem. Harbering et al. (2015) considers the problem which is equivalent to a job shop scheduling with two routes. It is shown how to achieve a lower bound on the makespan when all operations of the job shop model have equal processing times.

3. DYNAMIC PROGRAMMING APPROACH

In this section we assume that the order in which the trains, constituting the set N_1, depart from Station 1 and the order in which the trains, comprising the set N_2, depart from Station 2 are known. These orders can be either specified by some properties of the objective function or determined by some factors outside the considered model, for example by trains priorities.

For example, consider the objective functions

$$L_{max}(\sigma) = \max_{i \in N}\{C_i(\sigma) - d_i\}, \tag{2}$$

and

$$\sum w_i C_i(\sigma) = \sum_{i \in N} w_i C_i(\sigma), \tag{3}$$

where, for each $i \in N$, d_i is the point in time by which it is desired to complete the journey of train i and w_i is a weight associated with train i. According to the following