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Jointly Optimising Prices for Primary and Multiple Ancillary Products

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Abstract: There are many industrial applications where attempts are made to sell ancillary products or service with a primary item For instance, a warranty is often offered as an ancillary product to those who buy the television sets (the primary item). This has become even more important in the airline industry where ancillary products such as baggage fees have become very important to the financial health of the industry. In this paper, the question of what price to charge for the primary and ancillary products will be investigated.

Keywords: inventory, ancillary services, pricing, revenue management

1. INTRODUCTION

Historically, ancillary products have often been offered in industry: warranties with electronic goods, extended warranties with car sales, customized finishes to cars, engraved monographs, parking fees associated with a sporting event, etc. Recently, the issue of ancillary pricing has gained notoriety due to current airline practices to charge for items that were typically included in the ticket price. Most airlines, for instance, now charge for a checked bag. Some, such as *Spirit Air* even charge for a carry-on bag. Indeed, an airline traveller can be faced with multiple ancillary fees: priority boarding fees, premium seating fees, bag fees, etc. In 2014, the total baggage fees on US Airlines was $3,350,000,000! In this paper, the question of how to balance the prices for the primary and ancillary products is considered.
Breuckner et al. (2013) and Nickolae et al. (2013) have an empirical analysis of the effect of baggage fees on air fares. Gallego et al. (2013) consider the effect that ancillary fees have on operational aspects. Shulman and Geng (2012) look at a duopoly game involving a base good and add on customers. Allon et al. (2011) approaches a related problem but with the approach that both the customers and firm's wealth should be taken into consideration. Odegaard and Wilson (2015) look at the case of three distinct groups of customers with different willingness to pay distributions and allow for one primary product and one ancillary product. Often such problems can be reformulated as newsvendor problems. Pricing in a newsvendor setting is also considered in Anderson et al. (2015) and Wilson and Anderson (2015). Hamilton et al. (2010) consider the effect that "nickel and diming customers" might have. Total revenue to US airlines from US airlines has exploded over the last few years. (A good source of data in this can be found in http://www.rita.dot.gov/bts/sites/rita.dot.gov.bts/files/subject_areas/airline_information/baggage_fees/index.html)

Here the problem is formulated for a primary product and multiple ancillary products. The demand functions are assumed known. A general formulation is provided. Then, as is common in much of the literature, the focus is on linear demand functions. The case where demand functions are linear is often considered in the literature on pricing due to their mathematical tractability and the insight that such an analysis can bring. (However, in practice, one should be somewhat wary…see Wilson et al. (2011) for a discussion.) However, considering linear demand functions can be considered a first step in the analysis of the more complex problem of more applicable demand functions.

2. FORMULATION FOR MULTIPLE ANCILLARY PRODUCTS

Suppose there is one primary product and n-1 secondary products. Then the goal is to set the primary price p_1 and the secondary prices $p_2, \dots p_n$ to maximize expected revenue. We assume that there is a fixed capacity C of the primary product…e.g. seats on a flight or inventory of cars. As is common in much of the literature, it is assumed that products have already been paid for and the cost need not be considered in the analysis since ordering policies are not being considered,
Let $\{1,2, \dots J\}$ index the set of J distinct product combinations. For each $i \in \{1,2, \dots J\}$, let $n(i)$ denote the number of products and $i_1, \dots i_{n(i)}$ the actual products in the mix. Denote the demand function for product mix i by $g_i(p_{i_1} + \cdots + p_{i_{n(i)}})$. Note this allows individuals to buy different product mixes based on the price. For some prices, an individual may buy mix 1, for others mix 2, etc. For a given set of prices, $p_1, \dots, p_n$, the total demand is

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$\sum_{i=1}^{J} g_i(p_{i_1}+\cdots+p_{i_{n(i)}})$. Thus if the total demand is less than the availability C of the primary product, the revenue is $\sum_{i=1}^{J}(p_{i_1}+\cdots+p_{i_{n(i)}})g_i(p_{i_1}+\cdots+p_{i_{n(i)}})$. If the total demand is greater than C, then then revenue depends on the order in which customers arrive. For this case the mount sold at each price is now random and expected values will be derived. For instance if all the customers who want only the primary product arrive first, then there is limited (if any) opportunity to obtain revenue from some of the secondary products. Assume that customers arrive randomly. Then a revenue of $p_{i_1}+\cdots+p_{i_{n(i)}}$ will be received from exactly x customers with probability equal to

$$\frac{\binom{g_i(p_{i_1}+\cdots+p_{i_{n(i)}})}{x}\binom{-g_i(p_{i_1}+\cdots+p_{i_{n(i)}})+\sum_{i=1}^{J} g_i(p_{i_1}+\cdots+p_{i_{n(i)}})}{C-x}}{\binom{\sum_{i=1}^{J} g_i(p_{i_1}+\cdots+p_{i_{n(i)}})}{C}}$$

since x must come from $g_i(p_{i_1}+\cdots+p_{i_{n(i)}})$, the total demand for group i, and $C-x$ from the remaining demands. Thus the expected revenue from product mix i equals $C\frac{g_i(p_{i_1}+\cdots+p_{i_{n(i)}})}{\sum_{i=1}^{J} g_i(p_{i_1}+\cdots+p_{i_{n(i)}})}$.

Thus, for a given set of prices, $p_1,\ldots p_n$, the expected revenue is

$\sum_{i=1}^{J}(p_{i_1}+\cdots+p_{i_{n(i)}})g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)$ for $\sum_{i=1}^{J} g_i(p_{i_1}+\cdots+p_{i_{n(i)}})\le \mathrm{C}$,

and

$\frac{C}{\sum_{i=1}^{J} g_i(p_{i_1}+\cdots+p_{i_{n(i)}})}\sum_{i=1}^{J}(p_{i_1}+\cdots+p_{i_{n(i)}})g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)$ for $\sum_{i=1}^{J} g_i(p_{i_1}+\cdots+p_{i_{n(i)}})>\mathrm{C}$

This immediately suggests that only two solutions need be entertained: Find the prices $p_1,\ldots p_n$ that maximize $\sum_{i=1}^{J}(p_{i_1}+\cdots+p_{i_{n(i)}})g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)$. If the resulting demand does not exceed C, then this is the optimal solution. If the resulting demand is less than C, then the optimal prices maximize $\frac{\sum_{i=1}^{J}(p_{i_1}+\cdots+p_{i_{n(i)}})g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)}{\sum_{i=1}^{J} g_i(p_{i_1}+\cdots+p_{i_{n(i)}})}$.
Note that distinguishing between these two cases is the only place that the capacity C is considered.

3. LINEAR DEMAND FUNCTIONS

The case where demand functions are linear is often first considered in the literature due to their mathematical tractability. So this is the first step in the analysis of this problem. Specifically, assume that $g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)=a_i+b_i(p_{i_1}+\cdots+p_{i_{n(i)}})$ for each $i\in\{1,2,\ldots J\}$.

3.1First Order Conditions: Capacity at Least Equal to Demand

For each $i\in\{1,\ldots,n\}$ representing product I, let K_i denote the number of distinct product combinations that contains that product. So, for instance, for i=1, $K_1=J$ since the primary product is in all possible product combinations. Let R_i be the subset of $\{1,2,\ldots J\}$that contains the product mix identifiers that contain product i. Then the first derivatives are given by the following:

$$\frac{\partial}{\partial p_i}\left\{\sum_{i=1}^{J}(p_{i_1}+\cdots+p_{i_{n(i)}})g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)\right\}=\sum_{j\in R_i}a_j+2p_i\sum_{j\in R_i}b_j+2\sum_{j\in R_i}\sum_{k\in\{j_1,\ldots,j_{n(j)}\}\backslash i}b_jp_{j_k}$$

The above linear system of equations equals zero when, for each I,

$$p_i=\frac{-\sum_{j\in R_i}a_j-2\sum_{j\in R_i}\sum_{k\in\{j_1,\ldots,j_{n(j)}\}\backslash i}b_jp_{j_k}}{2\sum_{j\in R_i}b_j}$$

which are easy to solve via successive elimination as will be illustrated in the next section.
If $\sum_{i=1}^{J}(p_{i_1}+\cdots+p_{i_{n(i)}})g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)\le C$, at the solution to the above equations, then this solution is the global optimum.

3.2 First Order Conditions: Capacity Smaller than Demand

Now suppose that $\sum_{i=1}^{J}(p_{i_1}+\cdots+p_{i_{n(i)}})g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)>C$ for the solution in the last section.
At the solution to the above equations. Then it is necessary to maximise $\frac{\sum_{i=1}^{J}(p_{i_1}+\cdots+p_{i_{n(i)}})g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)}{\sum_{i=1}^{J} g_i(p_{i_1}+\cdots+p_{i_{n(i)}})}$. The first derivatives of this equal zero when

$$\sum_{i=1}^{J}g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)\frac{\partial}{\partial p_i}\left\{\sum_{i=1}^{J}(p_{i_1}+\cdots+p_{i_{n(i)}})g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)\right\}-\sum_{i=1}^{J}(p_{i_1}+\cdots+p_{i_{n(i)}})g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)\frac{\partial}{\partial p_i}\{\sum_{i=1}^{J}g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)\}=0$$

for each $i\in\{1,2,\ldots n\}$. It is necessary to write these derivatives with the p_i isolated to facilitate solution. First note, using the linear definition of the demand function that the first factor is linear in p_i for each i:

$$\sum_{i=1}^{J}g_i(p_{i_1}+\cdots+p_{i_{n(i)}})=\sum_{i=1}^{J}\left\{\sum_{j\in R_i}a_j+p_i\sum_{j\in R_i}b_j+\sum_{j\in R_i}\sum_{k\in\{j_1,\ldots,j_{n(j)}\}\backslash i}b_jp_{j_k}\right\}.$$

This is multiplied by a derivative which is also linear in p_i:

$$\frac{\partial}{\partial p_i}\left\{\sum_{i=1}^{J}(p_{i_1}+\cdots+p_{i_{n(i)}})g_i\left(p_{i_1}+\cdots+p_{i_{n(i)}}\right)\right\}=\sum_{j\in R_i}a_j+2p_i\sum_{j\in R_i}b_j+2\sum_{j\in R_i}\sum_{k\in\{j_1,\ldots,j_{n(j)}\}\backslash i}b_jp_{j_k}.$$

The first term after the subtraction sign is quadratic in p_i: