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Analysis of the Lead Time Distribution in Closed Loop Manufacturing Systems

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Abstract: In several manufacturing systems found in the automotive, food and semiconductor industries, product quality or value deterioration due to excessive residence time in the system are observed. This phenomenon generates defective or low value products, thus undermining the performance of these systems. In this paper, we develop an exact analytical method for calculating the lead time distribution of closed loop systems and present an heuristic to predict the peaks of this distribution. Numerical results show previously uninvestigated behavior of closed loop manufacturing systems under lead time constraints and provide insights on the design of these complex systems.

Keywords: Manufacturing systems, Lead time distribution, Perishable products, Loop systems

1. INTRODUCTION AND OBJECTIVES

The analysis of the lead time distribution is of paramount importance in manufacturing systems with product quality or value deterioration or in systems where strict lead time requirements are imposed by the market. Product quality and value deterioration due to excessive residence times, or lead times, during production is a significant phenomenon in several industries, including automotive, food manufacturing, semiconductor, electronics manufacturing and in polymer forming. For example, in automotive paint shops a car body that is affected by prolonged exposure to the air in the shop floor caused by excessive lead times between operations, is prone to particle contamination, leading to unacceptable quality of the output of the painting process. Moreover, food production is pervaded by strict requirements on hygiene and delivery precision requiring a maximum allowed storage time before packaging (Wang et al., 2014). If the production lead-time exceeds this limit, the product has to be considered as defective and cannot be delivered to the customer. In these systems, higher inventory increases the system throughput but also increases the production lead times, thus increasing the probability of producing defective items. Therefore, a relevant trade-off is generated between production logistics and quality performance that requires advanced system engineering methods to be profitably addressed (Colledani et al., 2014b), (Inman et al., 2013). The same situation is found in production systems where strict lead time constraints are imposed by the customers (Biller et al., 2013).

The first model considering this phenomenon is proposed in (Liberopoulos and Tsarouhas, 2002). The installation of a properly sized in-process buffer led to a reduction in failure impact on product quality and an increase of the system efficiency in a croissant production line. In (Liberopoulos et al., 2007), the authors focused on the production rate of asynchronous production lines in which long failures cause the material under processing in the upstream machines to be scrapped by the system. Moreover, in (Biller et al., 2013) raw material release policies have been proposed to maximize the throughput under an average lead time constraint. In these contributions, the analysis is focused on the average lead time and the distribution of lead time is not taken into consideration. More recently, the calculation of the distribution of the residence time in manufacturing systems have attracted increasing attention. In (Shi and Gershwin, 2012) a procedure to numerically compute the distribution of the lead time in two-machine lines with machines having one operational state and one down state has been proposed. In (Colledani et al., 2014a) the analysis of manufacturing systems under lead time dependent product deterioration has been proposed for two-machine lines with general Markovian machines. Finally, the performance of serial lines with product deterioration is analyzed by calculating the distribution of the residence time in Bernoulli lines in (Naebulharam and Zhang, 2014).

In this paper, we propose for the first time an exact analytical method for the calculation of the lead time distribution in closed-loop systems. Closed loop systems are typically found in situations where the workpiece needs to be clamped on a pallet before being processed by the machines in the system. After the loading process, the part on the pallet is processed by a finite number of processing stages and is released as a finished product by unloading the part from the pallet at the last processing

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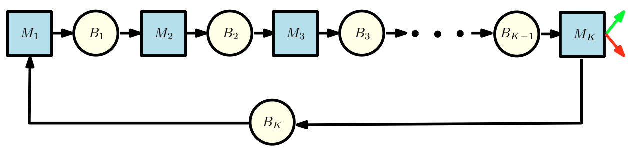

Fig. 1. Example of closed-loop systems.

station. As a consequence, a fixed and constant number of pallets circulates in the system. The same behavior is found in pull-type serial manufacturing systems operating under token-based production control policies, such as kanban, conwip and basestock. In this case, the number of tokens regulating the production flow in the system is fixed and constant due to demand constraints and new parts are accepted in the system only if free tokens are available. Both situation can be suitably analyzed by the method proposed in this paper. Moreover, in this paper general Markovian machines are considered, thus enabling to model a wide set of machine behaviors, including machines with multiple down states, machines with generally distributed operational and failure times, machines with correlated failure mechanisms, and stages with non-identical processing times. This feature makes the proposed approach applicable to a wide set of real manufacturing systems. The lead time distribution is used to calculate the throughput of parts respecting a given lead time constraint in systems subject to product deterioration or to lead time requirements imposed by the customer.

The paper is organized as follows. In section 2, the main assumptions of the analyzed set of manufacturing systems are introduced. In section 3, the exact analytical method used to calculate the lead time distribution is presented. In section 4, numerical results are provided and discussed. In section 5, insights on the structure of the lead time distribution are derived, with implications on the system design rules. Conclusions are drawn in section 6.

2. SYSTEM MODELING

2.1 System Architectures

We consider manufacturing systems formed by K machines and K buffers that form a closed-loop system, as represented in Figure 1. The material flow is modeled as a discrete flow of parts and the Blocking Before Service (BBS) mechanism is considered. Each buffer $B_i, i = 1, ..., K$, located between machine M_i and machine $M_{i+1 \pmod K}$, has finite capacity denoted as $N_i, i = 1, ..., K$. The number of pallets circulating in the system N_p is invariant.

2.2 Machine Behavior

The dynamics of each stage is modeled by a discrete-time and discrete-state Markov chain of general complexity. In detail, each stage $M_i, i = 1, ..., K$ is represented by I_i states, and thus the state indicator a_i assumes values in $[1, ..., I_i]$. The set containing all the states of M_i is called S_i. The dynamics of each stage in visiting its states is captured by the transition probability matrix λ_i, that is a square matrix of size I_i. Moreover, we use a vector that indicates for each state of M_i if in the state M_i is up or down. This vector is denoted by μ_i and its jth entry $\mu_{i,j}$ is equal to one (zero) if M_i is up (down) in state j. The set of up states of M_i is denoted as U_i and the set of down states is denoted as D_i. Stages with the same features have been considered for the first time in (Gershwin and Fallah-Fini, 2007), and, later, in (Colledani, 2013).

2.3 Part Quality or Value Deterioration

The quality or value of parts deteriorates with the time parts spend in a *critical portion of the system*, denoted by two integers, e and q with $1 \leq e \leq q \leq K$, and composed of those buffers that are between stages M_e and M_q, in the direction of the material flow. We will refer to as lead time of a part the time spent in the buffers between stages M_e and M_q. If $e = q$, then the whole cycle time of a part is considered. The probability that a part is defective is a non-decreasing function of its lead time. The function $\gamma(h)$ indicates the probability that a part released by the system is defective given that it spent h time units in the critical portion of system. Defective parts are scrapped at the end of the line.

2.4 Performance Measures

The main performance measures of interest for this set of systems are:

- Average total production rate of the system, denoted by E^{Tot}.
- Probability that the lead time, LT, is equal to a given number of time units, h, i.e, $P(LT = h)$.
- Average effective production rate of conforming parts, E^{Eff}, which is given by:

$$E^{Eff} = E^{Tot} \sum_{h=1}^{\infty} P(LT = h)[1 - \gamma(h)] \qquad (1)$$

- System yield, Y^{system}, i.e., fraction of conforming parts: $(E^{Eff} \ / \ E^{Tot})$.

3. LEAD TIME DISTRIBUTION

Since all machines are described by a discrete time Markov chain (DTMC) and the changes in the buffer levels are determined following the Blocking Before Service assumption, the overall behavior of the system can be modeled by a DTMC. In the following we describe this DTMC and exploit its modified version to determine the lead time distribution. For what concerns the state probability vectors of the involved DTMCs, the ith entry of a vector v will be denoted by $v(i)$.

3.1 Discrete time Markov chain of the system

The state of the system is given by a vector, $s = (a_1, ..., a_K, b_1, ..., b_K)$, that contains an entry for each machine, namely, the state of the machine, and an entry for each buffer, namely, the number of parts of the buffer. The entries $(a_1, ..., a_K)$ can assume values in $\prod_i^K I_i$ ways. The entries $(b_1, ..., b_K)$ can assume values in as many ways as many ways there exist to distribute the parts among the buffers. Let us denote the number of possibilities for $(b_1, ..., b_K)$ by C. If every buffer is large enough to