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Evaluation of a Power System Stability Based on Usage of Virtual Digital Analyzer

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Abstract. For early detection of the threat of a cascading failure we need to use the immune information system, the main subsystem of which is a virtual stability analyzer. We propose to implement the analyzer by usage of discrete linearized models of energy systems and apparatus of Lyapunov direct method, based on the study of discrete Lyapunov equation solutions, called Gramians. The functional of stability loss risk is equal to the square of H_2 - norm of the matrix transfer function, which is proportional to energy of the system. We propose to use a functional risk assessment through spectral decomposition of controllability and observability Gramians for evaluating of the energy accumulated in combinational dominant eigenvalues of the system. In comparison with the method of modal analysis the proposed approach allows to evaluate the impact of synergies between weak-stable eigenvalues and their combinations to the system stability loss risk and to detect the potential source of stability loss.

Keyword: stability loss risk, power system, discrete Lyapunov equation, controllability and observability Gramians, H_2-norm, matrix transfer function.

1. INTRODUCTION

The problem of the small signal analysis and power system monitoring of local or district oscillations is known in the world of science for over 50 years and has not yet lost its relevance (Kundur, 2005; CIEE, 2010). The researches in this field concerned with the direct Lyapunov method, modal analysis and research of the eigenvalues of the characteristic equation of the EPS mathematical model's matrix (Barquin, et al., 2012; Lyapunov, 1934). Many powerful researches were used, however, the practical value of these studies is largely limited, primarily due to the fact that the mathematical models of electric power systems are very sophisticated: they are mainly non-linear, non-stationary and distributed ones, including a variety of time scales. The most difficult problems arise due as to the high dimension of the power systems mathematical, as to the need to solve problems of high dimensionality in real time (Ahmetzyanov et al., 2012). The measuring methods are the following: Prony analysis, robust recursive least squares method, the Yule-Walker algorithms, wavelet analysis, neural networks and genetic algorithms (CIEE, 2010; Rehtans, 2003; Gagliotti et al., 2011) So, the determination of the static stability of a power grid the paper presents a Gramian method based on a new mathematical technique of solving Lyapunov and Silvester differential and algebraic equations which developed in the Institute of Control Sciences for the analysis of a stability degree of the linear dynamic systems (Yadykin, 2010; Yadykin et al., 2014). The method operates by means of decomposition of the Gramian matrix to make a solution of the Lyapunov or Sylvester equation into a spectrum expansion of matrices that make up these equations. The method allows to prescribe the criterion of Energy Power Systems (EPS) stability loss risk and also to evaluate the interaction of illstable system modes (Grobovoi et al., 2013). For the development of the immune system it is necessary to create a virtual analyzer (soft sensor) of the stability loss threat. There are several approaches to the design of such analyzer:

- Ill-stable oscillation power measurement (CIEE, 2010; Gaglioti et al., 2011),
- Selective modal analysis (Barquin, 2012)
- Modified Arnoldi method (Martins, 1997)
- Parametric identification methods (Bakhtadze, 2008)

For the soft sensor design one can use direct Lyapunov method, based on analysis of solution of the Lyapunov differential or algebraic equation in frequency domain (Yadykin et al., 2014).

In this work a new method is developed for the discrete matrix Lyapunov equation solving. The main functions of the system include: detection of a place of occurrence and severity of the threat; localization and isolation of the threat; preventive adaptive management and control; reconfiguration of a management system.

The system may also be implemented by the following features of the implementation:

- reception, processing, archiving and reporting (agent monitoring of the environment state and the internal state of the power system using the knowledge base);
- creating of a reference model of the current regime (agent forming a predictive reference model);

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- identification of a dangerous deviation from the projected state reference model and its localization (Agent forecast dangerous state energy system);
- forecast of the potential emergency and early discovery of the stability loss threat.

This approach was further developed in the paper (Yadkin et al., 2014) towards the use of a predictive model in virtual sensor device associative search and development of a risk assessment method. The method is based on the calculation discrete system Gramians, which structure and the parameters were obtained by the associative search method Bahtadze et al, 2008; Bahtadze et al., 2011). Gramians handling for discrete systems is proposed to calculate it for continuous system equivalent obtained from a discrete system by using a bilinear transformation. In this paper, the problem solution is proposed as a method of direct calculating of Gramians for discrete linear control systems.

2. THE PROBLEM STATEMENT

In the context of increasing demands for reliability and quality of energy supply and for the development of energy generation technologies, along with the growing deterioration of the main process equipment and distribution network, the effective solution of the problems of EPS monitoring and control is to create smart grid systems .The architecture of the control system becomes modular, interoperable and extensible. It is based on the use of sub-network cluster of multi-agent systems (MAS) with a vertical integration, employing the intelligent autonomous agents for various applications.

Creating a smart grid with advanced functions of the immune system will require grand investments. But the ultimate benefit for consumers, energy companies, government and society will provide a multiple return on investment, including increased reliability, security, survivability, power quality and reduce a pressure on the environment.

Let suppose, that a mathematical model of the power system is defined by the nonlinear algebraic-differential equations system (Kundur, 2005)

$$\begin{aligned}&\dot{\mathbf{x}}(t)=\mathbf{f}(\mathbf{x},\mathbf{u},t),\quad \mathbf{x}(t_0)=0,\\&\mathbf{M}(\mathbf{x},t)\mathbf{x}(t)=\mathbf{N}(\mathbf{x},t)\mathbf{u}(t).\end{aligned}$$

Linearized model in small relative to the power system fixed regime is defined as the linear algebraic-differential equations system

$$\begin{aligned}&\dot{\mathbf{x}}(t)=\mathbf{A}_1\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t),\quad \mathbf{x}(t_0)=0,\\&\mathbf{M}\mathbf{x}(t)=\mathbf{N}\mathbf{u}(t).\end{aligned}\tag{1}$$

Suppose that matrix **M** is a nonsingular one. Then the equations system (1) may be transformed as

$$\begin{aligned}&\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t),\quad \mathbf{x}(t_0)=0,\\&\mathbf{A}=\mathbf{A}_1+\mathbf{M}^{-1}\mathbf{N},\end{aligned}\tag{2}$$

Consider a stable linear discrete stationary dynamic system, obtained from (2) (Polyak and Shcherbakov, 2002; Poznyak, 2008)

$$\begin{aligned}&\mathbf{x}(k+1)=\mathbf{A}\mathbf{x}(k)+\mathbf{B}\mathbf{u}(k),\quad \mathbf{x}(0)=0,\\&\mathbf{y}(k)=\mathbf{C}\mathbf{x}(k)\end{aligned}\tag{3}$$

where $\mathbf{x}\in R^n,\mathbf{u}\in R^m,\mathbf{y}\in R^m$. We shall consider complex-value matrices of finite dimensions $\mathbf{A}_{[n\times n]},\mathbf{B}_{[n\times m]},\mathbf{C}_{[m\times n]}$, where *m, n* are any positive integers $m\le n$ .Let the system (3) is stable, fully observable and controllable and all eigenvalues of the matrix A are different. The system (3) characteristics in frequency domain are defined by the transfer function (Wilkinson, 1965)

$$\mathbf{H}(z)=\frac{\mathbf{M}_{n-1}z^{n-1}+...+\mathbf{M}_1z+\mathbf{M}_0}{N(z)},\quad \mathbf{M}_j=\mathbf{C}\mathbf{A}_j\mathbf{B}\tag{4}$$

where $\mathbf{A_i}$ is Faddeev matrix. The transfer function (4) is strictly proper. Consider the dual discrete algebraic Lyapunov equations associated with equation (1) of the form

$$\begin{aligned}&\mathbf{A}\mathbf{P}^c\mathbf{A}^*+\mathbf{B}\mathbf{B}^*=\mathbf{P}^c.\\&\mathbf{A}^*\mathbf{P}^o\mathbf{A}+\mathbf{C}^*\mathbf{C}=\mathbf{P}^o.\end{aligned}\tag{5}$$

It is well known, matrix **A** resolvent expansion has the form

$$(\mathbf{I}z-\mathbf{A})^{-1}=\sum_{j=0}^{n-1}z^j\mathbf{A}_j\times N^{-1}(z),\ (\mathbf{I}z-\mathbf{A}^*)^{-1}=$$
$$=\sum_{j=0}^{n-1}z^j\mathbf{A}^*{}_j\times N^{-1}(z),$$

where $N(s)$ – characteristic polynomial of matrix **A**. Matrices A_j are the Faddeev matrices (Faddeev and Faddeeva, 1963). Let $a_n=1,\mathbf{A}_{n-1}=\mathbf{I}$. Then Faddeev-Leverje algorithm for matrices A_j calculation may be written as (Kwakernaak and Sivan, 1972):

$$a_{n-k}=-\frac{1}{k}\mathrm{tr}(\mathbf{A}\mathbf{A}_{n-k}),\quad \mathbf{A}_{n-k-1}=-a_{n-k}\mathbf{I}+\mathbf{A}\mathbf{A}_{n-k},$$
$$k=1,2,...n.$$

The equations (5) solutions in time domain have the form (Antoulas, 2005)

$$\mathbf{P}^c=\sum_{k=0}^{\infty}\mathbf{A}^k\mathbf{B}\mathbf{B}^*(\mathbf{A}^*)^k,\quad \mathbf{P}^o=\sum_{k=0}^{\infty}(\mathbf{A}^*)^k\mathbf{C}^*\mathbf{C}\mathbf{A}^k.$$

The solutions in frequency domain have the form (Godunov, 1998; Demidenko, 2009)