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Deadlock-free scheduling for flexible manufacturing systems using untimed Petri nets and model predictive control

Dimitri Lefebvre

GREAH – Normandie Univ, UNIHAVRE, 75 rue Bellot, FR 76600 Le Havre
e-mail: dimitri.lefebvre@univ-lehavre.fr

Abstract: This paper proposes algorithms that incrementally compute control sequences that drive the marking of untimed Petri nets from an initial value to a reference one, avoiding forbidden states with a length or duration that approaches the minimal value. The proposed algorithms are applicable to a large class of discrete event systems with or without temporal specifications in particular in the domain of flexible manufacturing, communication and computer science or transportation and traffic. To overcome the most burdensome part of the computations, the sequences encoded in a small area of the reachability graph are worked out. The main contribution is to propose an estimation of the minimal length or duration of the remaining sequences to the reference based on the computation of the firing count vectors. The approach is suitable for deadlock-free scheduling problems encountered with flexible manufacturing systems.

Keywords: Discrete event systems, Petri nets deadlock-free scheduling problems, model predictive control.

1. Introduction

Petri nets (PNs) are commonly used for control issues of discrete event systems (DESs) (Cassandras, 1993; David & Alla, 1992) The design of firing sequences that reach a specific state and optimize a specific cost function (usually the makespan) is a basic objective in many control problems, in particular in deadlock-free scheduling problems because this objective leads directly to the design of optimal cycle of tasks in job shop problems. The difficulty is that deadlock-free scheduling problems are known to be NP-hard due to multi-layer resource sharing and routing flexibility of the jobs. Thus, a large literature has been devoted to such optimization, in particular with PN models. On the one hand, the main contributions to that problem have been provided by some adaptations of the Dijkstra and A* algorithms to the PN (Chretienne, 1986; Lee & DiCesare, 1994). Such algorithms are suitable to partially explores the reachability graph of the net, and generate efficient schedules with respect to a heuristic cost function. The performance basically lies in how good the cost function is. Numerous improvements have been developed: pruning of non-promising branches (Sun et al., 1994; Reyes-Moro et al., 2002), backtracking limitation (Xiong & Zhou, 1998), determination of lower bounds for the makespan (Jeng & Chen, 1998), best first search with backtracking and heuristic (Wang Q & Z, 2012) or dynamic programming (Zhang et al., 2005). On the other hand, some contributions are issued from the supervisory control methods that have been developed (Ramadge & Wonham, 1987; Basile et al., 2013) in order to avoid forbidden markings, in particular deadlock markings. Supervisory control is mainly based on the addition of generalized mutual exclusion constraints over the initial unconstrained PN model and these constraints are usually implemented with monitor places. A few results also combine scheduling and supervisory control in the same approach: search in the partial reachability graph (Lei et al., 2014), genetic algorithms (Abdallah et al., 2002) and heuristic functions based on the firing vector (Jeng & Chen, 1998) have been studied. But a common drawback of all these approaches is that they require a large computational effort and are time consuming. They are definitively not suitable for real time control or reconfiguration.

This work takes place in this context. It proposes a method for untimed PNs that incrementally computes deadlock-free schedules with control sequences that reach the reference state from the initial one, avoiding the forbidden states with a length or duration that is minimal or approaches the minimal length or duration. The method is inspired from model predictive control (MPC) approach. As a consequence, it is robust to perturbations induced by the firing of uncontrollable transitions because the control actions to apply are updated at each step. The approach is based on a partial exploration of the PN reachability graph but limits this exploration to the neighborhood of the current marking. For that purpose a criterion based on the evaluation of the number of firings or duration required to reach the reference from the final nodes of the graph is combined with a criterion that computes exactly the number of firings or the duration to reach the final nodes from the root of the graph. The present work improves our previous contributions in untimed (Lefebvre & Leclercq, 2015) context, where the path to the reference was computed according to an Euclidean distance.

The paper is organized as follows. In Section 2, PN systems and control problems are introduced. Section 3 presents an algorithm to approach minimal length firing sequences. Section 4 sums up the conclusions and perspectives.

2. Petri net systems

2.1. Petri nets

A PN structure is defined as $G = <\boldsymbol{P}, \boldsymbol{T}, W_{PR}, W_{PO}>$, where $\boldsymbol{P} = \{P_1,\ldots, P_n\}$ is a set of n places and $\boldsymbol{T} = \{T_1,\ldots, T_q\}$ is a set of q transitions of labels $\{1,\ldots,q\}$, $W_{PO} \in (\mathbf{N})^{n\times q}$ and $W_{PR} \in (\mathbf{N})^{n\times q}$ are the post and pre incidence matrices ($\mathbf{N}$ is the set of non-negative integer numbers), and $W = W_{PO} - W_{PR}$ is the incidence matrix. $<G, M_I>$ is a PN system with initial marking M_I and $M \in (\mathbf{N})^n$ represents the PN marking vector. The enabling degree of transition T_j at marking M is given by $n_j(M)$:

$$n_j(M) = \min\{\lfloor m_k / w^{PR}_{kj} \rfloor : P_k \in {}^{\circ}\boldsymbol{T}_j\} \quad (1)$$

where ${}^{\circ}\boldsymbol{T}_j$ stands for the set of T_j upstream places, m_k is the marking of place P_k, w^{PR}_{kj} is the entry of matrix W_{PR} in row k and column j. A

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transition T_j is enabled at marking M if and only if (iff) $n_j(M) > 0$, this is denoted as $M\,[T_j >$. The amount in which T_j fires is an integer α that satisfies $0 < \alpha \leq n_j(M)$ and the marking varies according to $\Delta M = M' - M = \alpha.W(:, j)$, where $W(:, j)$ is the column j of incidence matrix. This is denoted by $M\,[T_j > M'$ or by $M' = M + \alpha.W.X_j$ where X_j is the firing count vector of transition T_j (David & Alla, 1992). An untimed firing sequence σ fired at marking M_I is defined as $\sigma = T(j_1)T(j_2)\ldots T(j_h)$ where $j_1,\ldots j_h$ are the labels of the transitions. $X(\sigma)$ is the firing count vector associated to σ, $|\sigma| = \|X(\sigma)\|_1 = h$ is the length of σ ($\| \,.\, \|_1$ stand for the 1-norm), and $\sigma = \varepsilon$ stands for the empty sequence. When σ and σ' are two sequences, $\sigma\,\sigma'$ stands for the concatenation of σ and σ'. The untimed firing sequence σ fired at M leads to the untimed marking trajectory (σ,M):

$$(\sigma,M) = M\,[T(j_1) > M(1)\ldots.\ M(h\text{-}1)\ [T(j_h) > M_{ref}. \tag{2}$$

where $M(1),...,M(h\text{-}1)$ are the intermediate markings and M_{ref} is the final marking. A marking M is said reachable from initial marking M_I if there exists a firing sequence σ such that (s.t.) $M_I\,[\sigma > M$ and σ is said feasible at M_I. $\boldsymbol{R}(G, M_I)$ is the set of all reachable markings from M_I.

2.3. Control design for PNs and TPNs

For control issues, the set of transitions $\boldsymbol{T}$ is divided into 2 disjoint subsets $\boldsymbol{T_C}$, and $\boldsymbol{T_{NC}}$ such that $\boldsymbol{T} = \boldsymbol{T_C} \cup \boldsymbol{T_{NC}}$. $\boldsymbol{T_C}$ is the subset of q_C controllable transitions, and $\boldsymbol{T_{NC}}$ the subset of q_{NC} uncontrollable transitions. The firing of controllable transitions that are enabled can be enforced or avoided by the controller whereas the firing of uncontrollable transitions cannot be driven by the controller. $\boldsymbol{R}(G_C, M_I) \subseteq \boldsymbol{R}(G, M_I)$ is the set of all reachable markings from initial marking M_I by firing only controllable transitions and $G_C = <\boldsymbol{P}, \boldsymbol{T_C}, W_{PR\_C}, W_{PO\_C}>$ where W_{PR_C}, W_{PO_C} are extracted from matrices W_{PR} and W_{PO} by considering only the controllable transitions. Depending on the control application, some forbidden markings may be also specified. For this purpose, the function $LEGAL$ is defined for any marking $M \in \boldsymbol{R}(G_C, M_I)$ as $LEGAL(M) = 0$ if the marking is forbidden else $LEGAL(M) = 1$ and $\boldsymbol{R_L}(G_C, M_I) \subseteq \boldsymbol{R}(G_C, M_I)$ is the set of markings that are legal. The objective of the proposed control design is to reach a reference marking $M_{ref} \in \boldsymbol{R_L}(G_C, M_I)$ starting from initial marking M_I with a trajectory of minimal length (for PNs) or minimal duration (for TPNs) that has no uncontrollable transition and that visits no forbidden marking.

3. Model predictive control for untimed Petri nets

The basic idea of MPC is to anticipate the evolution of the system in order to achieve the control objective. At each step, a cost function is minimized, possibly under some constraints. A sequence of control actions is obtained. The first action of this sequence is applied and the prediction starts again from the new state reached by the system (Richalet et al., 1978; Camacho & Bordons, 2007).

3.1. Cost function based on Euclidean distance

In our previous works, an adaptation of the Dijkstra algorithm has been used to compute the set of legal marking trajectories with minimal length H^* from M_I to M_{ref} when only controllable transitions are fired. This algorithm was based on a partial exploration of the PN reachability graph so that this subgraph rooted in M_I includes only legal markings, reaches M_{ref} and has the smallest depth (Lefebvre & Leclercq, 2015). The main difficulty with this algorithm is the exponential complexity with respect to (wrt) the depth of the exploration: the method is time consuming and not acceptable when the markings M_I and M_{ref} are far from each other. In order to overcome this difficulty the partial exploration of reachability graphs has been combined with a model predictive control (MPC) approach based on the minimization of the cost function J_{Eucl}:

$$J_{Eucl}(M,M_{ref}) = (M - M_{ref})^T.I_n.(M - M_{ref}) \tag{4}$$

that corresponds to the Euclidean distance from the marking M to the reference M_{ref} in marking space (I_n stands for the identity matrix of dimension n x n). Sufficient conditions have been stated so that the proposed method converges to the reference (Lefebvre & Leclercq, 2015) but, in comparison with the exhaustive search, optimality cannot be ensured and near minimal length sequences are obtained (instead of minimal ones). One reason that explains sub-optimality is that the Euclidean distance in marking space does not always explain how "far" two markings are in terms of firing sequences. Moreover with criterion (4), the marking may be attracted by local minima that are due to hill-climbing phases (i.e. phases during which the distance to reference necessarily increases before it decreases). In the next section, a more efficient cost function is proposed.

3.2. Cost function based on the remaining firing count vector

In this section, the criterion (4) is replaced by an estimation of the number of firings required to reach the reference M_{ref} from the marking M. This estimation is based on the computation of the firing count vector X that satisfies $M_{ref} - M = W.X$ according to an integer optimization problem. For that purpose let us first detail a transformation of the incidence matrix with Proposition 1:

Proposition 1: Let consider a PN structure $G = <\boldsymbol{P}, \boldsymbol{T}, W_{PR}, W_{PO}>$ with $W = W_{PO} - W_{PR} \in (\mathbf{Z})^{n\times q}$ ($\mathbf{Z}$ is the set of positive and negative integer numbers) of rank r. There exists a regular matrix $P_L \in (\mathbf{Z})^{n\times n}$ and a regular permutation matrix $P_R \in \{0,1\}^{q\times q}$ such that:

$$W' = P_L.W.P_R = \begin{pmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{pmatrix} \tag{5}$$

where $W_{11} = \in (\mathbf{Z})^{r\times r}$ is an integer regular (i.e. of rank r) upper triangular matrix, $W_{21} = 0_{(n-r)\times r}$ and $W_{22} = 0_{(n-r)\times(q-r)}$ (i.e. zero matrices of appropriate dimensions).

Proof: the proof consists in the iterative construction of the regular matrices $P_L \in (\mathbf{Z})^{n\times n}$ and $P_R \in \{0,1\}^{q\times q}$. Let $w^1_{min} = w_{i^*j^*}$ such that $|w_{i^*j^*}| = \min\,(|w_{ij}| : i = 1,\ldots,n, j = 1,\ldots,q)$. There exists two permutation matrices $P_{left1} \in \{0,1\}^{n\times n}$ and $P_{right1} \in \{0,1\}^{q\times q}$ such that w^1_{min} is placed in the upper left corner of matrix $P_{left1}.W.P_{right1}$. There also exists a regular diagonal matrix D_{left1}:

$$D_{left1} = \begin{pmatrix} sign(w^1{}_{min}) & 0 \\ 0 & I_{n-1} \end{pmatrix}$$

with sign(x) = 1 if x > 0, sign(x) = 0 if x = 0 and sign(x) = -1 if x < 0. Finally there exists a lower triangular regular matrix C_{left1}:

$$C_{left1} = \begin{pmatrix} \alpha_1 & 0 \\ X & I_{n-1} \end{pmatrix}$$

with $\alpha_1 > 0$, such that $W_1 = C_{left1}.D_{left1}.P_{left1}.W.P_{right1} = P_{L1}.W.P_{R1}$ with:

$$W_1 = \begin{pmatrix} W^1{}_{11} & W^1{}_{12} \\ W^1{}_{21} & W^1{}_{22} \end{pmatrix}$$

and $W^1{}_{11} = w^1{}_{min} \in (\mathbf{Z})^{1\times 1}$ of rank 1, $W^1{}_{12} = \in (\mathbf{Z})^{1\times(n-1)}$, $W^1{}_{21}$ is a zero matrix of dimensions (n-1)x1 and $W^1{}_{22} \in (\mathbf{Z})^{(n-1)\times(q-1)}$ is of rank r-1. The matrix $P_{L1} \in (\mathbf{Z})^{n\times n}$ is regular and $P_{R1} \in \{0,1\}^{q\times q}$ is a permutation matrix. The same transformation is repeated r times so that finally (5) holds with: