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State Estimation for DES according to Partially Observed Stochastic Petri Nets

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Abstract: This paper concerns state estimation of stochastic discrete event systems. For that purpose, partially observed stochastic Petri nets are used to model the system and the sensors. From the proposed modelling and the collected measurements, timed sequences which are consistent with those measurements are obtained. Based on the events date, our approach consists on evaluating the probabilities of the marking trajectories using probabilistic model. Such probabilities are important since they reflect the most probable behavior of the system. State estimation is obtained as a consequence.

Keywords: State estimation, stochastic Petri nets, probabilistic model.

1. INTRODUCTION

Discrete Event Systems (DESs) are the class of systems where the state evolution depends entirely on the occurrence of discrete events (Cassandras & Lafortune, 2008). DESs are usually used to represent and study manufacturing systems. For such systems, state estimation becomes an important issue for control and diagnosis problems when some events are not measurable and remain silent.

With untimed Petri Net (PNs) some contributions already exist for the state estimation problem. The basic problem is to determine which states are consistent with a given untimed observation sequence of ordered events. The pioneer works are about observation issues with DESs (Ramadge, 1986) and the design of observers for PNs (Ramirez-Treviño et al., 2000). Then, place/transition nets have been used to solve the problem of marking estimation in untimed context under the assumption that the net structure is known, the initial marking is not known, and all transition firings can be observed (Giua & Seatzu, 2002). Late, unobservable and indistinguishable transitions have been introduced, under the assumption that the initial marking is known (Cabasino et al., 2011). Recently, a probabilistic marking estimation is studied in (Cabasino et al., 2015) where *a priori* probabilities of the transition occurrences are assumed to be available. Some contributions have been proposed to relax the assumption about the measurement of initial marking in deterministic (Li & Hadjicostis, 2013) and probabilistic setting (Cabasino et al., 2014). Partial information about the marking are also considered (Lefebvre 2013, 2014b).

But, the explicit consideration of time is crucial for the specification and verification of many manufacturing systems. In the timed context, the problem is to determine the states that are consistent with a timed observation sequence (a sequence of events and dates of occurrence) collected in a given time interval. In the Timed PNs frameworks, contributions also exist for the state estimation problem depending on the temporal extensions that are considered. Two main classes of extensions are used: Time PNs where transitions may fire within given time intervals that are associated with places or transitions (Merlin, 1974) and Timed PNs where enabled transitions fire as soon as the given time delays have elapsed (Ramchandani, 1974).

For Time PNs, a state estimation method has been developed based on the state class graph for unlabeled nets without cycles of unobservable transitions (Ghazel et al., 2009). Improvements of this method have been developed in (Basile et al., 2013) with a modified state class graph and later in (Basile et al., 2015) where not only the estimation of the set of markings consistent with the observation, but also a series of time intervals, one for each enabled transition are characterized in linear algebraic terms. Based on marking observer, the state estimation has been also considered by solving linear programming problem to check the set of firing sequences feasible on the untimed underlying PN and consistent with the current observation (Bonhomme, 2013) (Declerck & Bonhomme, 2014). For Timed PNs, in particular Stochastic PNs (SPNs) few contributions exist because the dates of silent events are non-deterministic. The computation of the most probable date of events has been recently proposed for Partially Observed SPNs (POSPNs) (Ammour et al., 2015). This contribution takes place in this context and extends our previous result in order to compute the probabilities of states. The paper is organized as follows. In Section 2, POSPNs and timed observation sequences are described. In Section 3, a method to calculate the probability of marking trajectories is detailed. In Section 4, the state estimation problem is studied. Finally, some conclusions and perspectives are presented in Section 5.

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2. MODELING OF MARKOVIAN STOCHASTIC PROCESS WITH PETRI NETS

2.1. Partially observed stochastic Petri nets

PNs are graphical and mathematical modelling tools. In this paper, we consider SPNs which are characterized by random firing delays associated with the transitions (Molloy, 1982). Formally, a SPN structure is defined as $G_s = <P, T, W_{PR}, W_{PO}, \mu>$, where $P = \{P_1,\ldots, P_n\}$ is a set of n places and $T = \{T_1,\ldots, T_q\}$ is a set of q transitions, $W_{PO} \in (\mathbb{N})^{n\times q}$ and $W_{PR} \in (\mathbb{N})^{n\times q}$ are the post and pre incidence matrices ($\mathbb{N}$ is the set of non-negative integer numbers), and $W = W_{PO} - W_{PR}$ is the incidence matrix. $\mu = (\mu_j) \in (\mathbb{R}^+)^q$ ($\mathbb{R}$ is the set of real numbers) is the firing rate vector which characterizes the transition firing periods. $< G_s, M_I >$ is a SPN system with initial marking M_I and $M \in (\mathbb{N})^n$ represents the PN marking vector.

A transition T_j is enabled at marking M if and only if (iff) $M \geq W_{PR}(:, j)$, where $W_{PR}(:, j)$ is the column j of the pre incidence matrix; this is denoted as $M\ [T_j >$. For each transition T_j, enabled at marking M, the firing periods are given by a random variable with an exponential probability density function (pdf) of parameter $n_j(M).\mu_j$ where $n_j(M)$ stands for the enabled degree of transition T_j at marking M which is given by:

$$n_j(M) = \min\left(\left\lfloor\left(\frac{m_k}{w_{PR}(k,j)}\right)\right\rfloor \text{ such that } P_k \in {}^{\circ}T_j\right) \quad (1)$$

where ${}^{\circ}T_j$ is the set of T_j upstream places denoted as P_k and m_k their markings, $w_{PR}(k,j)$ is the element of row k and colomn j of matrix W_{PR}. Finally, $\lfloor . \rfloor$ stands for the lower rounded value of (.). When T_j is enabled, it may fire, and when T_j fires once, the marking varies according to $\Delta M = M' - M = W(:, j)$. This is denoted as $M\ [T_j > M'$.

Under some specific assumptions, the SPN behaves as a Markov model and its steady state is computed by solving a usual Chapman Kolmogorov equation (Molloy, 1982) (Bobbio et al., 1998) (Lefebvre, 2014a).

An untimed firing sequence σ_U of size $h = |\sigma_U|$ fired at marking M_I is a sequence of h transitions $\sigma_U = T(1)T(2)\ldots T(h)$, with $T(j) \in T$, $j = 1,\ldots,h$ that consecutively fire from M_I. This leads to the untimed marking trajectory (2):

$$(\sigma_U, M_I) = M(0)\ [T(1) > M(1)\ldots.[T(h) > M(h). \quad (2)$$

with $M(0) = M_I$. The integer $x_j(\sigma_U)$ is the number of occurrences of transition T_j in σ_U and $X(\sigma_U) = (x_j(\sigma_U)) \in (\mathbb{N})^q$ is the firing count vector for σ_U.

A marking M is said to be reachable from initial marking M_I if there exists a firing sequence σ_U such that $M_I\ [\sigma_U > M$. When time is considered, a timed firing sequence σ_T fired at marking M_I in time interval $[t_0, t_h]$ is defined in a similar way: $\sigma_T = T(t_1)T(t_2)\ldots T(t_h)$ where t_j, $j = 1,\ldots,h$ represents the firing date of transition $T(t_j) \in T$ that satisfy $t_0 \leq t_1 \leq t_2 \leq \ldots \leq t_h$. This leads to the timed marking trajectory (3):

$$(\sigma_T, M_I) = M(t_0)\ [T(t_1) > M(t_1)\ldots.[T(t_h) > M(t_h) \quad (3)$$

with $M(t_0) = M_I$.

In the case of POSPNs, a labeling function $\mathcal{L} : T \rightarrow E \cup \{\varepsilon\}$ is introduced that assigns a label to each transition where $E = \{e_1,\ldots,e_{qo}\}$ is the set of q_O labels that are assigned to observable transitions and ε is the null label that is assigned to the silent ones. The labeling function is extended to firing sequences and the concatenation of labels obviously satisfies: $\varepsilon.\varepsilon = \varepsilon$ and $\varepsilon.e_k = e_k.\varepsilon = e_k$. The function $\mathcal{L}$ is represented by the event sensor matrix $L = (l_{kj}) \in (\mathbb{N})^{qo \times q}$ such that $l_{kj} = 1$ if $L.X(T_j) = e_k$ else $l_{kj} = 0$. The marking sensor matrix $H \in (\mathbb{R})^{no \times n}$ defines the projection of the marking vector M over a subset of observed places of dimension n_O. The measured part of the marking is denoted as $M_O = H.M$, where $H = (h_{kj}) \in (\mathbb{N})^{no \times n}$ such that $h_{kj} = 1$ if the marking of the place p_i is considered in the k^{th} measure.

Definition 1: A POSPN system is defined as $<G_O, \mu, M_I>$ with $G_O = <G_s, L, H>$ where G_s is a SPN structure, L is the event sensor matrix, H is the marking sensor matrix, μ is a firing rate vector and M_I is the initial marking. The matrices L and H determine the sensor configuration.

2.2. Timed observation sequences

A measurement function Γ is introduced according to the sensor configuration. Measurements are collected over the time interval $[\tau_0, \tau_{end}]$. When the POSPN marking varies with the firing of a single transition T at date $t \in [\tau_0, \tau_{end}]$, $\Gamma(T(t), \tau_0, \tau_{end})$ is defined by (4):

$$\Gamma(T(t), \tau_0, \tau_{end}) = (H.M(\tau_0))\ L.X(T(t))\ (H.M(\tau_{end})) \text{ if } (H.M(\tau_0) \neq H.M(\tau_{end})) \vee (L.X(T(t)) \neq \varepsilon)$$
$$\Gamma(T(t), \tau_0, \tau_{end}) = (H.M(\tau_{end})) \text{ if } (H.M(\tau_0) = H.M(\tau_{end})) \wedge (L.X(T(t)) = \varepsilon) \quad (4)$$

Note that in (4), event and marking measurements are dated. The measurement function Γ is then extended to marking trajectories of the form (3) measured over the time interval $[\tau_0, \tau_{end}]$:

$$\Gamma((\sigma_T\ T(t), M_I), \tau_0, \tau_{end}) = \Gamma((\sigma_T, M_I), \tau_0, \tau_{end})\ L.X(T(t))\ (H.M(\tau_{end})) \text{ if } (H.M(t_h) \neq H.M(\tau_{end})) \vee (L.X(T(t)) \neq \varepsilon)$$
$$\Gamma((\sigma_T\ T(t), M_I), \tau_0, \tau_{end}) = \Gamma((\sigma_T, M_I), \tau_0, \tau_{end}) \text{ if } (H.M(t_h) = H.\ M(\tau_{end})) \wedge (L.X(T(t)) = \varepsilon) \quad (5)$$

We consider that the measurement function Γ collects the K successive dated marking and event measurements of a timed marking trajectory (σ_T, M_I) over time horizon $[\tau_0, \tau_K]$ (where τ_K is the date of the last measurement) and organizes them in the timed observation sequence (6):

$$TR_O(\tau_0, \tau_K) = \Gamma((\sigma_T, M_I), \tau_0, \tau_K) = M_O(\tau_0)\ e_O(\tau_1)\ M_O(\tau_1)\ e_O(\tau_2)\ \ldots\ M_O(\tau_{K-1})\ e_O(\tau_K)\ M_O(\tau_K) \quad (6)$$

where $M_O(\tau_0) = H.M_I$, K is the length of the observation sequence and τ_j, $j = 1,\ldots,K$ refer to the dates of