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Modelling production cost with the effects of learning and forgetting

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Abstract: Defining a dynamic model for calculating production cost is a challenging goal that requires a good fitting ability with real data over time. A novel cost curve is proposed here with the aim of incorporating both the learning and the forgetting phenomenon during both the production phases and the reworking operations. A single-product cost model is thus obtained, and a procedure for fitting the curve with real data is also introduced. Finally, this proposal is validated on a benchmark dataset in terms of mean square error.

Keywords: learning curves; production cost; autonomous learning; induced learning; curve fitting

1. INTRODUCTION

Experience is a concept that extends across a large pool of disciplines. For centuries, people have known that *repetita iuvant* ("repeating does good"), but only in recent decades have researchers attempted to translate this aphorism into mathematical models. Several experience curves have therefore been proposed in scientific and humanistic branches and nowadays these play a crucial role in industrial and social fields.

In the industrial field, learning curves are certainly correlated with the strategic dimension. Several authors have highlighted that learning curves represent sources of competitive advantage (e.g. Hatch et al., 2004), which means that a firm with a steeper learning curve than its competitors may gain a competitive advantage in the long term. Furthermore, learning curves may also drive medium to short-term decisions along tactical and operative dimensions respectively. Ergonomics (e.g. Anzanello and Fogliatto, 2011), assembly and production lines (e.g. Anzanello and Fogliatto, 2007; Anzanello and Fogliatto, 2010; Dolgui et al., 2012; Jaber and Glock, 2013; Otto and Otto, 2014; Pan et al., 2014), inventory control (e.g. Jaber and Guiffrida, 2008; Jaber et al., 2010; Teng et al., 2014; Khan et al., 2014), production planning (e.g. Glock et al., 2012) and quality improvement (e.g. Lolli et al., 2016) are some examples of the application of learning curves in different research fields.

Indeed, many learning phenomena may also simultaneously concur with a single dependent variable, as is evident in the case of total costs accounting such as total production cost. A complete, reliable and (ideally) easy-to-use total production cost model is certainly a challenging goal, and is necessarily related to model and fit with more simultaneous learning processes. The core of the present paper is thus to propose a total cost production model which embraces autonomous, induced and forgetting components, both in production and in reworking activities. In particular, reworking activities are considered here for the first time as affected by a learning phenomenon. Moreover, the dual source of experience, i.e. autonomous and induced, enriches the accounting model with the aim of making it both more flexible and suitable for application to several optimisation problems involving the proactive intervention of management. Induced learning activities such as training and investment strategies for improved productivity and quality should in fact be supported by a cost model that takes into account both sources of experience.

A four-parameter curve is achieved and a fitting procedure is proposed for establishing these parameters. To the best of the authors' knowledge, this is new in the field of learning curves and is assumed to have promising applications.

The remainder of the paper is organized as follows. Section 2 contains the notation adopted throughout the manuscript. Section 3 focuses on some relevant contributions in the field of learning curves. Section 4 presents the cost model, while Section 5 is devoted to the procedure for fitting the curve parameters. Section 6 reports the experimental analysis, and Section 7 closes the paper with some conclusions and the further research agenda.

2. NOTATION

c_{in}^{p} = initial unitary cost of production.
$\bar{c}_{i}^{f.int}$ = initial unitary cost of internal failure of type i.
$c_{i}^{f.ext}$ = unitary cost of external failure of type i.
c_{min}^{p} = minimum initial unitary cost of production.
$c_{s}^{p}(t)$ = minimum unitary cost of production in period t.
$\hat{c}_{i}^{f.int}(t)$ = minimum unitary cost of internal failure of type i in period t.
$Q_{cum}(t)$ = cumulative volume of production in period t.
$q_{cum}(t)$ = cumulative volume of failures in period t.

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$Q(t)$ = production volume in period t.
$q(t)$ = number of failures in period t.
Φ_i = likelihood of failure of type i.
$q_i(t) = q(t) \cdot \Phi_i$ = number of failures of type i in period t.
p_i = likelihood that a failure of type i is internal.
$(1 - p_i)$ = likelihood that a failure of type i is external.
$\hat{Q}(t) = Q_{cum}(t) - q_{cum}(t)$ = number of conforming products produced until period t.
$c^p(t)$ = unitary production cost in period t.
$c_i^{f.int}(t)$ = unitary cost of failure of type i in period t.
K = form factor of autonomous learning.
$\beta(t)$ = improvement factor in period t.
C_{TOT} = total cost (production cost + failure cost + prevention cost + appraisal cost).
α = parameter of forgetting in production.
γ = parameter of forgetting in reworking.
$t_{stop}^p(t)$ = number of consecutive periods without production until period t.
$t_{stop}^f(t)$ = number of consecutive periods without reworks for failure of type i until period t.
$P(t)$ = prevention cost in period t.
$A(t)$ = appraisal costs in period t.
$h(t)$ = number of training hours in period t.
c_{tr} = hourly training cost.
$C_m(t)$ = preventive maintenance cost in period t.
c_a = unitary appraisal cost.
$C^f(t)$ = failure cost in period t.

3. RESEARCH BACKGROUND

An early attempt to define a relationship between production volume and performance increase was made by Wright (1936) who introduced a mathematical model describing how an increase in performance is related to an increase in the production rate. The Wright learning curve is represented by the following log-linear model:

$$C_{TOT} = c_{in}^p \cdot x^b \qquad (1),$$

where x is the number of units to produce and b represents the slope of the learning curve.

In subsequent years, a lot of alternative models were presented which may be clustered into two different categories depending on the number of independent variables, i.e. univariate and multivariate. The univariate models can in turn be clustered into three categories, i.e. log-linear, exponential and hyperbolic. For an in-depth explanation of these categories of learning models, the reader is referred to Anzanello and Fogliatto (2011) and Grosse et al. (2015).

Wright's model contains some significant drawbacks:

- If cumulative production goes to infinity, Wright's model is unreliable because it does not show any plateau effect, i.e. the total cost goes to zero if cumulative production tends to infinity. This is not possible because of fixed costs (Jaber and Glock, 2013);
- It supposes that production is defect-free, which is unrealistic (Jaber and Glock, 2013);
- It does not consider the forgetting component. In many processes the forgetting evaluation may be as important as the learning one;
- It deals only with autonomous learning, i.e. learning-by-doing. The induced component is not taken into account despite its relevance as competitive leverage;
- The prior experience in a task is neglected.

Many models have therefore been proposed for improving Wright's model. In particular, many models aim to solve the problem of prior experience, the most famous of which is the Stanford-B model:

$$C_{TOT} = c_{in}^p \cdot (x + B)^b \qquad (2),$$

where B is the number of units previously produced. Although this model improves Wright's model, it still retains all the other drawbacks.

Another model has been specifically proposed with the goal of introducing the plateau effect; this consists of adding a constant (lower bound) to Wright's model as follows:

$$C_{TOT} = c_{min}^p + c_{in}^p \cdot x^b \qquad (3)$$

In this case, if cumulative production goes to infinity, the total cost tends to c_{min}^p. However, this model still has all the other drawbacks of the original approach.

One of the first attempts to model the forgetting phenomenon was made by Carlson and Rowe (1976), who created a forgetting curve similar to Wright's learning curve. This approach was validated some years later by Globerson et al. (1989), whose empirical finding is that the log-linear model describes better than others both the workers' forgetting and the learning phenomenon. Carlson's forgetting model is as follows:

$$\hat{C}_{TOT} = \hat{c}_{in}^p \cdot x^f \qquad (4),$$

where $\hat{C}_{TOT}$ is the cost for the xth unit of lost experience of the forgetting curve, $\hat{c}_{in}^p$ is the cost for the first unit of the forgetting curve, x is the number of units that would have been produced if production had not stopped, and f is the slope of the forgetting curve. Alternative forgetting curves have been proposed by Jaber and Bonney (1996) and Tarakci et al. (2013). In particular, the former integrates Wright's learning curve with Carlson's forgetting curve, leading to the first learn-forget curve, but the other drawbacks still remain.

For a long time most of the proposed models retained the strong hypothesis of defect-free processes. The quality-based element in the learning curves was firstly introduced by Jaber and Guiffrida (2004), who proposed two different cases. The first one extends Wright's law with the hypothesis that the process is not defect-free, but the workers do not learn by reworking:

$$C_{TOT} = c_{in}^p \cdot x^b + c_i^f \cdot \rho \cdot x \qquad (5),$$

where ρ is the likelihood that a process goes out of control and c_i^f is the unitary failure cost. Conversely, the second case allows the workers to learn by reworking as follows:

$$C_{TOT} = c_{in}^p \cdot x^b + 2c_i^f \cdot \left(\frac{\rho}{2}\right)^{1-\varepsilon} \cdot x^{1-2\varepsilon} \qquad (6),$$