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Integrated Single Item Lot-Sizing and Quality Inspection Planning

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Abstract: This paper proposes an integrated model for Single Item Dynamic Lot-Sizing (SIDLS) problem and Quality Inspection Planning (QIP). The objective is to provide a model of production planning that takes into account a targeted level of outgoing quality (AQL: Acceptable Quality Level) when the manufacturing system inherently generates a proportion of defectives that increases significantly when the system switches from the in-control state to the out-of-control state. The Average Outgoing Quality (AOQ) of each period of time of the planning horizon is bounded as a function of the inspection capacity. The effects of integrating quality inspection planning are analyzed and discussed through several experiments representing different quality control system's parameters, i.e. inspection capacity, inspection cost and AQL.

Keywords: Lot-Sizing, Quality Assurance; Inspection Capacity; Average Outgoing Quality.

1. INTRODUCTION

Improving quality helps companies to increase their market share. This can be seen in practice and was shown through surveys with companies (See Keller and Noori (1988)). Quality control activities have a direct link with other activities in the company such as inventory control. Still, research on integrated decisions including quality control and inventory control are quite recent and just started in the mid-1980s with the publications of Lee and Rosenblatt (1985); Rosenblatt and Lee (1986) and Porteus (1986). These publications and others showed the importance of considering quality issues and process imperfection in inventory control policies/decisions. Actually, the greatest interest in integrated quality and inventory control models took place in the last two decades (Andriolo et al. (2014)).

The recent surveys (Wright and Mehrez (1998) and Khan et al. (2011)) about extended inventory control models with imperfect quality witness the increasing interest in this topic and its applications such as wafers fabrication (Lee (1992)) and paper production (Moisio and Virolainen (1993)). However, in all these surveys and based on our search, there is no research work on the integration of dynamic lot sizing models with imperfect quality products. This is the research gap that we attempting to close in this work.

Dynamic lot sizing problems are production planning problems in which decisions are taken over a planning horizon of T time periods. Demands are dynamic and can vary from one period to another. In each time period, the quantities to be produced and inventory levels have to be determined. The most common objective is to minimize the total cost composed of setup, production, and inventory holding costs. The most basic dynamic lot sizing problem is the single item problem studied by Wagner and Whitin (1958). Many extensions were considered later to include production and inventory capacity constraints, backlogging, multiple product types, setup times, multiple production levels, multiple resources, etc. There is a very rich literature about the dynamic lot sizing problems and surveys can be found in Karimi et al. (2003), Brahimi et al. (2006), and Díaz-Madroñero et al. (2014). In our work, we limit the discussion to the single item lot sizing problem with production capacity.

It seems that Lee and Rosenblatt (1985); Rosenblatt and Lee (1986) and Porteus (1986) are among the first to publish on importance of the relationship between quality and lot sizing. Porteus (1986) developed a deterministic model to demonstrate the importance relationship between quality and lot size. They present a closed form solution to the optimal lot size quantity given that the production process can go out of control with a certain probability and that there is a cost associated with repairing each defective unit of the product. The paper concluded that the optimal strategy consists of a combination in quality improvement investment and setup cost reduction (to reduce lot sizes). Keller and Noori (1988) extended Porteus work by considering that demand during lead time is probabilistic. They proposed a four-step solution procedure to solve a lot-size reorder-point model.

Porteus (1986) and Rosenblatt and Lee (1986) consider that the process is checked in the beginning of any new lot to ensure that it starts in an "in-control" state. If the process is in-control when it starts processing an item, it will either switch to out-of-control or continue in the in-control state after the production of the item. Once the process switches to out-of-control state, it will stay in that state and keep producing defective items till all items in the lot are produced. Khouja (2005) and later Jaber

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(2006) reformulated the model of Porteus (1986) to allow process adjustment (interruption) within a production cycle (lot). The idea of Khoudjas formulation might be motivated by the "andon" system in JIT philosophy which empowers operators to stop the process if something goes wrong (Jaber (2006)).

Porteus (1986) considers that all items produced in the in-control state are conform and all those produced in the out-of-control state are non-conform. Djamaludin et al. (1994) assumes that only a fraction of products are non-conform in each state. There are two types of non-conforming items: Type-A and Type-B. While, items of type A are non-operational, items of type B are operational but have performance characteristics which are inferior to those of conforming items (See Djamaludin et al. (1994)). While type-A items must be fixed or replaced, items of type B can be sold to the customer as they are at the risk of having to replace them later for the customer as part of a free replacement warranty (FRW) policy (Djamaludin et al. (1994)). Djamaludin et al. (1994) and Yeh and Chen (2006) consider items of type B nonconformity.

With respect to previous studies, we are considering that there are non-conform items both in in-control and out of control states at different probabilities. We are also limiting our study to class A problems only, that is, non-conform items are non-operational. Finally, replacement of non-conform items is assumed to instantaneous.

The remainder of the paper is organized as follows. Section 2 will present the model and the linear programming formulation. Section 3 presents numerical tests and discussions of the obtained results. Finally, a conclusion and some research perspectives are presented in Section 4.

2. PROBLEM FORMULATION

The Problem under consideration in this work concerns the joint planning of production and quality inspection. The aim of this study is to highlight the interaction of these two activities at the tactical decision making level of production control. The following notations are used:

T	production horizon, i.e. number of planning periods
t	index of planning period, $t \in \{1, 2, .., T\}$
c_t^p	unit production cost at period t
c_t^s	setup cost at period t
c_t^h	unit holding cost per time period at period t
c_t^i	unit inspection cost at period t
d_t	demand at period t
X_t	quantity to produce at period t
X_t'	quantity to sample at period t
X_t''	quantity to deliver without inspection at period t
Y_t	binary decision variable $= 1$ if production is to be set up at period t; 0 otherwise.
Z_t	quantity to inspect at period t
I_t	quantity to stock at period t
I_t'	quantity of sampled products to stock at period t
I_t''	quantity of inspected products to stock at period t
R_t	expected quantity of reworked items that are produced at period t (see eq.10)
W_t	state of requirement satisfaction at period t, i.e. $W_t = 1$ if $OQ_t > AQL$; 0 otherwise
OQ_t	expected proportion of defectives delivered at period t
AQL	acceptable quality level, i.e. proportion of defectives beyond which the delivery's quality is considered unsatisfactory
L	inspection lead time
p_0	proportion of defectives when the process is in-control
p_1	proportion of defectives when the process is ou-of-control, with $p_1 = \delta p_0$, where δ is the drift magnitude
$\lambda(t)$	probability of drift after processing the t-th item

The flow model and the associated decision variables are sketched out in Figure 1. Four levels of decision making are distinguished. The first level corresponds to the decisions related to production, i.e. the quantities to produce and quantities to stock. At the second level, the decisions concern the sampling of products to be inspected in the current period or later, that is why this model assumes the existence of a stock of sampled products. In the inspection level, the decisions concern the quantity of products to inspect and to stock. This model proposes the possibility to take into account an inspection lead time L. In this paper, the latter is neglected ($L = 0$).

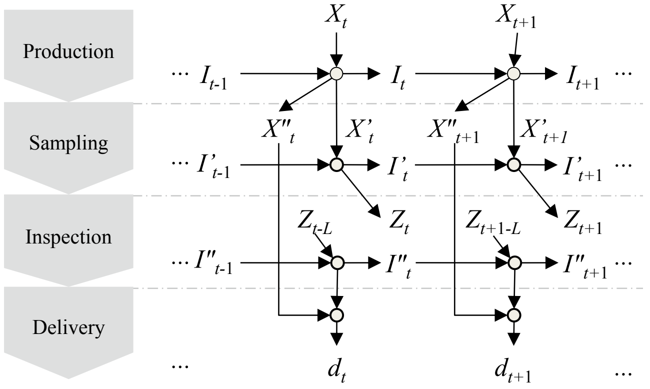

Fig. 1. Flow model and decision variables

The model presented in Figure 1 is used to evaluate and to analysis a common quality assurance strategy. This strategy is based on a specific agreement between the customer and manufacturer which specifies that each lot delivered to customer with a proportion of defectives that exceeds AQL is returned to manufacturer, who inspects