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A mixed-integer programming model for integrated production and maintenance

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Abstract: This paper deals with the production planning and preventive maintenance scheduling on a single machine multi-product capacitated lot-sizing problem (CLSP). The machine is assumed to be subject to random failures. Preventive maintenances at the beginning of each production period to reduce the risk of failure and minimal repairs at failure is considered. The aim is to minimize the sum of the total production and maintenance costs related to inventory, backorder, production, set-up, preventive maintenance (PM), and minimal repair (MR) under demand satisfaction and machine capacities constraints over the entire horizon. Using the Weibull model, we present a mixed-integer linear programming (MIP) model to solve the problem.

Keywords:
capacitated lot-sizing problem, production planning, preventive maintenance, minimal repair, Mixed-integer linear programming.

1. INTRODUCTION

The CLSP problem is one of the well-known models at the tactical level. Since it takes into account the constraint of system capacity, this problem has been proven to be NP-hard even without setup times by Florian et al. (1980) and Bitran and Yanasse (1982). Consequently, Chen and Thizy (1990) have shown that the multi-item CLSP problem, is strongly NP-hard. J et al. (1991) have shown that even finding a feasible solution for CLSP with setup times is NP-hard.

Budai et al. (2008) reviewed the majority of integrated maintenance and production models and subdivided the research area into four categories: high level models, economic manufacturing quantity (EMQ) models, deteriorating production system with buffer capacity (e.g., Rezg et al. (2004); Chelbi et al. (2008)) and finally, production and maintenance optimization rate (e.g., Kenné and Gharbi (2004); Gharbi and Kenné (2005); Dong-Ping (2009)).

Aghezzaf et al. (2007) present a model to determine an integrated production and maintenance planning for a production system, subject to random failures. At each maintenance intervention, the production system is not available, which decreases its production capacity. The purpose is to find the best lot size and PM cycle that satisfied the demand for all items over the entire horizon without backlogging while minimizing the expected sum of production and maintenance costs. Numerical example was used to illustrate the superiority of joint decision-making. Aghezzaf and Najid (2008) extended the model to parallel production lines system with both cyclical and non-cyclical PM policies and suggested an approximate algorithm based on Lagrangian decomposition to solve the problem. In the two articles mentioned above, backorder is not allowed. Najid et al. (2011) extended the prior model of Aghezzaf et al. (2007) into an integrated model for a single machine system with PM actions carried out in time windows and allowance of demand shortage when capacity is not sufficient to meet all demand. Recently, Wang (2013) extended the model of Aghezzaf et al. (2007) to a more complete and flexible model with cyclical PM policy and minimal repair at failure. Yildirim and Nezami (2014) analyzed an integrated maintenance and production planning with energy consumption and minimal repair. The authors considered that the processing times are affected by machine degradation. Fitouhi and Nourelfath (2012) consider also that the capacity is affected by machine degradation. Yildirim and Nezami (2014) studied the impact of reliability on energy consumption by adding the energy cost in the objective function. Our contribution can be easily applied for the study of Yildirim and Nezami (2014). The proposed non-linear model can be linearized using the same techniques proposed in this paper. The advantage to linearize the model is the opportunity to solve a larger problem with multi-products and multi-periods case at optimal by using commercial softwares or exact methods. Zhao et al. (2014)

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proposed the integrated model of production planning and maintenance. The authors propose an iterative approach to avoid the non-linearity of the maintenance cost.

2. INTEGRATING MAINTENANCE AND PRODUCTION DECISIONS

Preventive maintenance and production are mutually in conflict for two main reasons. First, since the time taken by PM activities could be used for production, production managers usually fail to realize the importance of PM. Second, delaying PM for production may increase the probability of failures, while maintenance managers try to reach high equipment availability. In practice, production and maintenance planning activities are usually performed independently. Therefore, it cannot be guaranteed that the obtained plans are optimal with respect to the objective minimizing the total maintenance and production cost. Better solutions could be obtained when maintenance planning is integrated with manufacturing activities. The integration of PM and production decisions may reduce not only the interruption time, but the total expected cost also. Depending on the production environments, this integration could be done either at the scheduling operational level (short-term) or at the tactical level (mid-term). The time horizons may vary for each planning level depending on the industry. Typical values are one week or less for operational planning, and one month or more for tactical planning.

For equipment that is not highly reliable, PM schedules may be weekly or even daily. In these environments, it is necessary to use a job-to-job PM planning tool. This issue has been dealt with in Cassady and Kutanoglu (2005), where the authors have developed a model integrating job sequencing and PM decisions. Considering that preventive maintenance, and repair affect both available production time, and the probability of machine failure, the model in Cassady and Kutanoglu (2005) coordinates PM planning decisions with single-machine scheduling decisions so that the total expected weighted completion time of jobs is minimized.

3. STATEMENT OF THE PROBLEM

The notations are presented in Table 1. Consider a single machine system that produces a set products P ($p \in P$) during a finite horizon of T ($t \in T$) periods. Each product that has a deterministic demand d_{pt} would be satisfied at the end of a period t. The machine has a production capacity for each period based on its age. Setup is carried out while changing from one product to another. The machine is subject to random failures. Scheduled PM can be performed according to the "as good as new" policy (perfect maintenance) and MR is carried out to restore the machine to an "as bad as old" condition without affecting its age. We assume that a PM is performed at the beginning of the first period ($t = 1$) and that there is no initial inventory or backorder of any product. The objective is to determine jointly the optimal production and maintenance plans which yield a minimal total cost.

Table 1. Notations

Notation	Definition
Indexes	
p	product index
t	planning period index
Parameters	
P	number of products
H	planning horizon
T	number of periods in a planning horizon
τ	length of a production planning period t
h_{pt}	holding cost per unit of p at the end of period t
b_{pt}	backorder cost per unit of p at the end of period t
π_{pt}	production cost per unit p in period t
s_{pt}	set-up cost of producing product p in period t
d_{pt}	demand of p at the end of period t
g	machine nominal production rate
a_t	the effective age of the machine at the beginning of period t
Z	a maintenance policy for the machine during the planning horizon
$G_t(Z)$	machine production rate at the beginning of period t for the Z maintenance policy
$CM(Z)$	total maintenance cost for the Z maintenance policy
$M[0,x[$	expected number of failures during $[0,x[$
CMR	minimal repair cost of the machine
CPM	preventive maintenance cost of the machine
TMR	expected minimal repair time of the machine
TPM	expected preventive maintenance time of the machine
Decision Variables	
I_{pt}	inventory level of product p at the end of period t
B_{pt}	backorder level of product p at the end of period t
x_{pt}	quantity of product p produced in period t
y_{pt}	binary variable that is equal to 1 if the setup of p occurs in period t, and 0 otherwise
z_t	binary variable is equal to 1 if a PR action is performed at the beginning of period t, 0 otherwise

4. SOLUTION METHOD

In order to show the performance of our contribution, we describe first the previous work of Fitouhi and Nourelfath (2012) to treat the problem. Then we present a new way to integrate a Weibull distribution as a machine lifetime in a linear mathematical model. And finally a MIP model is proposed to solve efficiently the problem.

4.1 Fitouhi and Nourelfath Work

Evaluating the maintenance cost and the production capacity reduction The expected number of failures during $[0,x[$ is as follows:

$$M[0,x[= \int_0^x r(y)\mathrm{d}y \tag{1}$$

where $r(y)$ is the failure rate of the machine obtained from its probability density function $f(y)$ as follows:

$$r(y) = \frac{f(y)}{1 - F(y)} \tag{2}$$

According to the assumptions of minimal repair and perfect PM the effective age of the machine is as follows:

$$a_t = (1 - z_t) \cdot (a_{t-1} + \tau) \qquad \forall t = 1, \cdots, T \tag{3}$$