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Design of a functional adaptive observer for bilinear delayed systems

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Abstract: A Functional Adaptive Observer (FAO) for a bilinear system with multiple discrete delays, is proposed in this paper. This observer allow the simultaneous estimation of the states and the unknown parameter. Necessary and sufficient conditions of unbiasedness of the functional observer are given. Stability conditions, which depend on system's delays, are given in terms of Linear Matrix Inequalities (LMIs) by using a polytopic LPV approach.

Keywords: Adaptive algorithms, Lyapunov function, Linear Matrix Inequality (LMI), Time delay systems, Bilinear systems.

1. INTRODUCTION

The control or observation problem of systems with delay are of both theoretical and practical importance (see Yan et al. (2012), Ezzine et al. (2011), Kadhraoui et al. (2015), Trinh (1999), Briat et al. (2011) and the references therein for example). Time delay exists in most of real industrial processes and can be a source of instability. Even a small delay can affect considerably the performance of a system.

In this note, we consider the presence of time delays, as well as the presence of unknown parameter in the class of bilinear systems. So the difference between Briat et al. (2011) and this paper is that we consider adaptive observer (presence of a constant parameter to estimate) and multiple delays in the state of the system to estimate. Another difference is that here we consider a descriptor approach which permits us to transform the problem into LMI by the use of a polytopic approach.
For such a problem, the adaptive observer may be a solution. It allows a simultaneous estimation of the states and the unknown parameter, even when time delays appear. The objective is to design an observer such that the state estimation error and the parameter estimation error converge both to zero. This algorithm is an active field of research in the recent years for nonlinear systems Besançon (2000), Zhao et al. (2012), Ibrir (2009), Choi et al. (2003).

In this work, the stability conditions of the observer depend on the time delays of the systems. The stability is studied by introducing a descriptor system and using a Lyapunov-Krasovskii function. The control signal is also considered bounded and as varying parameter. These conditions are expressed via matrix inequalities, which present some bilinearities. To overcome this problem, a Linear parameter Varying (LPV) approach for descriptor systems is used to lead to Linear Matrix Inequalities (LMI) which are more tractable. This approach has been used successfully in the recent years in Gérard et al. (2010), Mohammadpour and Scherer (2012) in the case without delay.

This paper is organized as follow. In section II, the formulation of the problem is given. Then, the unbiasedness conditions of the adaptive functional observer are presented in section III. Section IV is devoted to the delay-dependent stability analysis of the proposed observer using LMIs formulation.

2. PROBLEM STATEMENT

This note is devoted to the design of a full order functional adaptive observer for a bilinear system with multiple discrete delays. The proposed observer allows a suitable simultaneous estimation of the states and the unknown parameters using the inputs and outputs measurements. Thus, we consider the following system:

$$\begin{aligned}\dot{x}(t) &= A_0 x(t) + \sum_{i=1}^{m} A_i u_i(t) x(t) + A_{d_0} x(t-\tau_0) \\ &\quad + \sum_{i=1}^{m} A_{d_i} u_i(t) x(t-\tau_i) + Bu(t) + G\theta \\ y(t) &= Cx(t) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $\theta \in \mathbb{R}^q$, $y \in \mathbb{R}^p$ are the state vector, the measured input, the constant unknown parameter and the measured output respectively. τ_j, $j = 0, \dots, m$, are known constant delays. The matrices: A_j, A_{d_j}, B, G, and C, $j = 0, \dots, m$, are known with constant values and with

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appropriate dimensions.
Assume that the input $u(t)$ is continuous and bounded such that $u(t) \in \mathcal{U} \subset \mathbb{R}^m$, where

$$\mathcal{U} = \{u : t \to \mathbb{R}^m / \forall t \in \mathbb{R}^+, u_{i,\min}(t) \leq u_i(t) \leq u_{i,\max}(t), \\ \mu_{i,\min}(t) \leq \dot{u}_i(t) \leq \mu_{i,\max}(t)\} \quad (2)$$

The functional adaptive observer proposed has the following form:

$$\begin{aligned}
\dot{z}(t) &= N_0 z(t) + \sum_{i=1}^{m} N_i u_i z(t) + F u(t) + N_{d_0} z(t-\tau_0) \\
&\quad + \sum_{i=1}^{m} N_{d_i} u_i z(t-\tau_i) + M_0 y(t) + \sum_{i=1}^{m} M_i u_i y(t) \\
&\quad + M_{d_0} y(t-\tau_0) + \sum_{i=1}^{m} M_{d_i} u_i y(t-\tau_i) + TG\hat{\theta} \\
\dot{\hat{\theta}}(t) &= Q(y(t) - \hat{y}(t)) \\
\hat{x}(t) &= z(t) + Ey(t)
\end{aligned}$$

where $\hat{x}(t)$ and $\hat{\theta}(t)$ are respectively the estimates of $x(t)$ and θ. $z \in \mathbb{R}^n$ is the observer state and $\hat{y} = C\hat{x}(t)$. The matrices N_i, M_i, for $i = 0, \ldots, m, d_0, \ldots, d_m$, F, T, Q and E are unknown and should be determined such that the estimation error $e(t)$ given in (3) converges to zero.

3. UNBIASEDNESS CONDITIONS

Let us note e the estimation error, such that:

$$e(t) = \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix} = \begin{pmatrix} x(t) - \hat{x}(t) \\ \theta(t) - \hat{\theta}(t) \end{pmatrix} \quad (3)$$

We start by rewriting the error $e_1(t)$ as follows:

$$e_1(t) = x(t) - \hat{x}(t) = \Psi x(t) - z(t)$$

where $\Psi = I_n - EC$.
Putting that $T = \Psi$, then, the dynamics of the error e_1 is given by:

$$\begin{aligned}
\dot{e}_1(t) &= (N_0 + \sum_{i=1}^{m} N_i u_i(t)) e_1(t) + N_d e_1(t-\tau_0) \\
&\quad + \sum_{i=1}^{m} N_{d_i} u_i(t) e_1(t-\tau_i) + (F - \Psi B) u(t) \\
&\quad + \Psi G e_2(t) + (\Psi A_0 - N_0 \Psi - M_0 C) x(t) \\
&\quad + \sum_{i=1}^{m} (\Psi A_i - N_i \Psi - M_i C) u_i(t) x(t) \\
&\quad + (\Psi A_{d_0} - N_{d_0} \Psi - M_{d_0} C) x(t-\tau_0) \\
&\quad + \sum_{i=1}^{m} (\Psi A_{d_i} - N_{d_i} \Psi - M_{d_i} C) u_i(t) x(t-\tau_i) \quad (4)
\end{aligned}$$

On another side, the time derivative of e_2 is given by

$$\dot{e}_2(t) = \dot{\theta}(t) - \dot{\hat{\theta}}(t) = -QCe_1(t) \quad (5)$$

Thus, in order to ensure the unbiasedness of the FAO, the dynamics of the error estimation must be independant of the state x and the input u. Then, the matrix F must be chosen as:

$$F = \Psi B \quad (6)$$

and the following Sylvester equations must hold for $j = 0, \ldots, m$:

$$\Psi A_j - N_j \Psi - M_j C = 0 \quad (7a)$$
$$\Psi A_{d_j} - N_{d_j} \Psi - M_{d_j} C = 0 \quad (7b)$$

For the resolution of the Sylvester equation, we develop the equations (7a) and (7b) as:

$$A_j = N_j + K_j C + ECA_j \quad (8a)$$
$$A_{d_j} = N_{d_j} + K_{d_j} C + ECA_{d_j} \quad (8b)$$

where

$$K_j = M_j - N_j E$$
$$K_{d_j} = M_{d_j} - N_{d_j} E$$

The equations in (8) can be rewritten in the following compact form

$$\beta = \chi \Lambda \quad (9)$$

where

$$\beta = [\,\overline{A}, \overline{A}_d]$$
$$\chi = [\,\overline{N}, \overline{N}_d, \overline{K}, \overline{K}_d, E]$$
$$\Lambda^T = \begin{pmatrix} I_{n(m+1)} & 0 \\ 0 & I_{n(m+1)} \\ \overline{C} & 0 \\ 0 & \overline{C} \\ C\overline{A} & C\overline{A}_d \end{pmatrix}$$

with

$$\overline{A} = [A_0, \ldots, A_m], \overline{A}_d = [A_{d_0}, \ldots, A_{d_m}]$$
$$\overline{N} = [N_0, \ldots, N_m], \overline{N}_d = [N_{d_0}, \ldots, N_{d_m}]$$
$$\overline{K} = [K_0, \ldots, K_m], \overline{K}_d = [K_{d_0}, \ldots, K_{d_m}]$$
$$\overline{C} = \mathrm{diag}(C, \ldots, C)$$

where $\mathrm{diag}(A, B)$ is a block diagonal matrices with A and B as diagonal blocks.

Equation (9) admits a solution if and only if the following rank condition is satisfied

$$\mathrm{rank}(\Lambda) = \mathrm{rank}([\beta^T, \Lambda^T]^T) \quad (10)$$

Or, this condition is always satisfied due to the structure of matrix Λ. Indeed, Λ is a full row rank matrix with $\mathrm{rank}(\Lambda) = 2n(m+1)$.

By defining $\alpha = 2(m+1)(n+p) + p$, the general solution of (9) is given by

$$\chi = \beta \Lambda^{\dagger} + Z(I_\alpha - \Lambda \Lambda^{\dagger}) \quad (11)$$

where Z is an arbitrary matrix with appropriate dimension, which will be chosen in order to guarantee the convergence of the estimation error, and $\Lambda^{\dagger}$ is any generalised inverse of Λ, which fulfil the following equation Rao and Mitra (1971):

$$\Lambda = \Lambda \Lambda^{\dagger} \Lambda \quad (12)$$

Let $[Z_1, Z_{d_1}, Z_2, Z_{d_2}, Z_3]$ be the partition of the matrix Z according to χ, and one can choose generalised inverse of Λ as follows:

$$(\Lambda^{\dagger})^T = \begin{pmatrix} I_{\tilde{n}} & 0_{\tilde{n}} \\ 0_{\tilde{n}} & I_{\tilde{n}} \\ 0_{\tilde{n}\times\tilde{p}} & 0_{\tilde{n}\times\tilde{p}} \\ 0_{\tilde{n}\times\tilde{p}} & 0_{\tilde{n}\times\tilde{p}} \\ 0_{\tilde{n}\times p} & 0_{\tilde{n}\times p} \end{pmatrix}$$

where $\tilde{n} = n(m+1)$ and $\tilde{p} = p(m+1)$.
So that, the observer matrices are given in (11) can be expressed as