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Standardizability of multivariable linear sampled-data systems with delay ⋆

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Abstract: Due to periodic sampling, linear sampled-data systems are a subclass of linear continuous periodic systems, even in the case, when all other elements are time invariant. Therefore, the well-established methods for LTI systems cannot be applied. Owing to the great practical importance of sampled-data control systems, various approaches for the rigorous description of those systems are known. Moreover, a lot of methods have been developed, that are able to yield rigorous solutions for numerous control and optimization problems. The application of those methods need representations in certain standard forms. Often, it is not clear, whether a given system can be transformed into a standard form or not. The paper considers this question for the standard sampled-data system with delay, for which numerous methods and tools are available, if the system belongs to the subclass with model structure. The paper provides necessary and sufficient conditions for a SD system with delay in standard structure to belong to this important subclass. The proof is given on basis of the parametric transfer matrix concept, which is at present the only one that can handle problems, where both difficulties - sampling and arbitrary time-delay - occur at the same time.

Keywords: Linear systems, Periodic sampling, Sampled-data systems, Time-delay, Transfer functions, MIMO systems

1. INTRODUCTION

Sampled-data (SD) systems are characterized by the presence of as well continuous as discrete-time elements. Due to the massive introduction of digital controllers and filters, SD systems are of great practical importance. In case of linear time-invariant processes and controllers, there exist two simple approaches for analysis and design of SD systems. The first one is called quasi-continuous approach. Here the system is considered as purely continuous system, and e.g. the controller is design as a continuous one. After that the continuous controller is substituted by a digital approximation, e.g. by Tustin's formula. The second approach is a pure discrete one. Here the sampled process is substituted by a discrete model. Then analysis and design are completely done in discrete time, e.g. by using z-transforms. However, in this case no information about the intersample behavior is provided. Detailed descriptions of both approaches can be found in standard textbooks, e.g. Åström and Wittenmark [1997], Franklin et al. [2002]. Nevertheless, both approaches are approximations, and already in simple cases the results can be unusable, Rosenwasser and Lampe [2000], Polyakov et al. [2002].

Therefore, since the beginning of the 1990th, new methods with exact and complete information have been developed for SD systems. This methods are referred under the concept of sampled-data in a stricter sense. However, due to the periodic sampling, a SD system is a continuous periodic system, and the standard methods for LTI systems are no longer applicable. Three approaches have been prepared for analysis and design of SD control systems. The lifting technique is based on state space and implies a transfer to representations with infinite input and output spaces, Bamieh and Pearson [1992]. Various control and optimization problems for SD systems have been solved by lifting, e.g. Chen and Francis [1995]. A corresponding approach in the frequency domain is called FR operator, Hagiwara and Araki [1995], which also leads to operations with infinite dimensional matrices. Due to the infinite dimension of the transformations, both approaches have problems with time-delay, which brings another infinite order. The third approach bases on the parametric transfer function (PTF) concept and it is applied in the present paper. The PTF is a generalization of the well-known ordinary transfer function for LTI systems to the case of linear time-varying systems. For MIMO SD systems it operates with parametric transfer matrices (PTM), but these matrices are of finite dimension. The concept has proved to solve manifold control problems for SD systems with delay, Lampe and Rosenwasser [2010, 2013, 2015], when the system is described as a certain standard model. The paper at hand investigates, whether a SD system with delay can be brought into this form.

2. SYSTEM DESCRIPTION AND PROBLEM

1) In Lampe and Rosenwasser [2010, 2013, 2015] the subject of investigation was the standard model of sampled-

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data (SD) system with delay, where the continuous process was described by the relations

$$\begin{aligned}\frac{\mathrm{d}v(t)}{\mathrm{d}t} &= Av(t) + B_1 x(t-\tau_1) + B_2 u(t-\tau_2),\\ y(t) &= C_2(t)v(t).\end{aligned} \tag{1}$$

Herein y is the process output vector, v is the process state vector, $x(t)$ is the vector of the external excitation, and $u(t)$ is the control vector. Moreover, A, B_1, B_2, C_2 are constant real matrices of appropriate size, and τ_1, τ_2 are nonnegative real constants.

Assume that process (1) is affected by a multivariable hold, described by

$$u(t) = h(t-kT)\psi_k, \quad kT < t < (k+1)T. \tag{2}$$

Herein, T is the sampling period, and $h(t)$ is a matrix, which determines the shape of the control impulses, and its entries are functions of bounded variation on the interval $0 \le t \le T$. Moreover, in (2) the quantity ψ_k is the vector control sequence, which is determined as solution of the difference equation

$$\alpha(\zeta)\psi_k = \beta(\zeta)y_k, \quad y_k = y(kT), \quad (k = 0, \pm 1, \ldots), \tag{3}$$

where

$$\begin{aligned}\alpha(\zeta) &= \alpha_0 + \alpha_1\zeta + \ldots + \alpha_\rho\zeta^\rho, \quad \det\alpha_0 \ne 0,\\ \beta(\zeta) &= \beta_0 + \beta_1\zeta + \ldots + \beta_\rho\zeta^\rho\end{aligned} \tag{4}$$

are polynomial matrices in ζ - the backward shift operator, Åström and Wittenmark [1997], with the effect

$$\zeta\psi_k = \psi_{k-1}, \quad \zeta y_k = y_{k-1}. \tag{5}$$

Assume that the matrices $\alpha(\zeta)$ and $\beta(\zeta)$ are left coprime in the sense of Kailath [1980].

As output of the SD system, we consider

$$z(t) = C_1 v(t-\tau_3) + Du(t-\tau_2-\tau_3), \tag{6}$$

where C_1, D are constant matrices, and τ_3 is a real constant, which also can take negative values.

The totality of equations (1) - (6) define the multivariable SD system with delay $\mathcal{S}_\tau$, which is called standard model of SD system with delay.

2) As was stated in Lampe and Rosenwasser [2010], when applying the Lapalce transformation, the standard model $\mathcal{S}_\tau$ can be described by the equations with the operator $p \triangleq \frac{\mathrm{d}}{\mathrm{d}t}$

$$\begin{aligned}z(t) &= K_\tau(p)x(t) + L_\tau(p)u(t),\\ y(t) &= M_\tau(p)x(t) + N_\tau(p)u(t),\end{aligned} \tag{7}$$

where

$$\begin{aligned}K_\tau(p) &= K(p)\,\mathrm{e}^{-p\tau_K}, \quad L_\tau(p) = L(p)\,\mathrm{e}^{-p\tau_L},\\ M_\tau(p) &= M(p)\,\mathrm{e}^{-p\tau_M}, \quad N_\tau(p) = N(p)\,\mathrm{e}^{-p\tau_N}.\end{aligned} \tag{8}$$

The matrices $K(p), L(p), M(p), N(p)$ are rational matrices, determined by the relations

$$\begin{aligned}K(p) &= C_1(pI_\chi - A)^{-1}B_1,\\ L(p) &= C_1(pI_\chi - A)^{-1}B_2 + D,\\ M(p) &= C_2(pI_\chi - A)^{-1}B_1,\\ N(p) &= C_2(pI_\chi - A)^{-1}B_2,\end{aligned} \tag{9}$$

where I_χ is the $\chi \times \chi$ identity matrix. Moreover, in (8) $\tau_K, \tau_L, \tau_M, \tau_N$ are constants obtained by the relations

$$\begin{aligned}\tau_K &= \tau_1 + \tau_3, \quad \tau_L = \tau_2 + \tau_3,\\ \tau_M &= \tau_1, \quad \tau_N = \tau_2.\end{aligned} \tag{10}$$

When we interprete (2) - (4) as a computer, the standard model $\mathcal{S}_\tau$ can be configured to the scheme shown in Fig. 1, which is called standard structure.

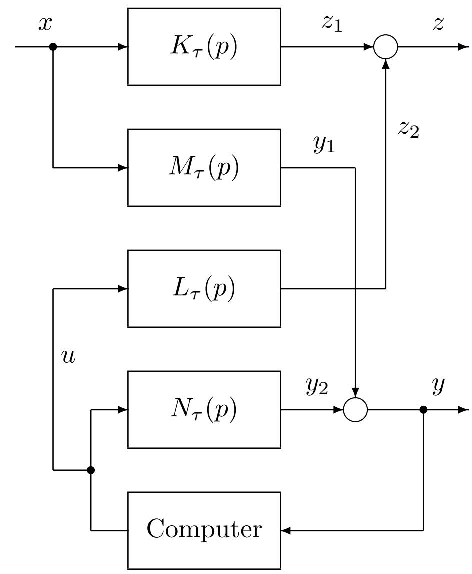

Fig. 1. SD system with delay in standard structure

3) As was shown in Lampe and Rosenwasser [2010], that under certain restrictions, which are fulfilled in paractical applications, for $x(t) = \mathrm{e}^{\lambda t} I$, where λ is a complex parameter, for all λ excluding a certain countable set, there exists a unique matrix solution of equations (1) - (6), in which

$$\begin{aligned}y(\lambda,t) &= \mathrm{e}^{\lambda t}\,W_{yx}(\lambda,t), \quad W_{yx}(\lambda,t) = W_{yx}(\lambda,t+T),\\ v(\lambda,t) &= \mathrm{e}^{\lambda t}\,W_{vx}(\lambda,t), \quad W_{vx}(\lambda,t) = W_{vx}(\lambda,t+T),\\ u(\lambda,t) &= \mathrm{e}^{\lambda t}\,W_{ux}(\lambda,t), \quad W_{ux}(\lambda,t) = W_{ux}(\lambda,t+T),\\ z(\lambda,t) &= \mathrm{e}^{\lambda t}\,W_{zx}(\lambda,t), \quad W_{zx}(\lambda,t) = W_{zx}(\lambda,t+T),\\ \psi_k(\lambda) &= \mathrm{e}^{\lambda T}\,\psi_{k-1}(\lambda).\end{aligned} \tag{11}$$

Here $W_{yx}(\lambda,t)$, $W_{vx}(\lambda,t)$, $W_{ux}(\lambda,t)$, $W_{zx}(\lambda,t)$ are matrices of suitable size. Below, these matrices are called parametric transfer matrices (PTM) from input $x(t)$ to the outputs y, v, u, z, respectively.

4) In Lampe and Rosenwasser [2010] it was shown that the PTM $W_{zx}(\lambda,t)$ has the form

$$W_{zx}(\lambda,t) = \mathrm{e}^{-\lambda\tau_L}\,\phi_{L\mu}(T,\lambda,t-\tau_L)\tilde{R}_N(\lambda)M_\tau(\lambda) + K_\tau(\lambda), \tag{12}$$