Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-10 (2016) 071–076

Electromechanical Delayed Resonator Implementation using Piezoelectric Networks

Ayhan S. Kammer*, Nejat Olgac**

University of Connecticut, Storrs, CT 06269 USA.
**Tel: 860-486-3567; e-mail: kammer@engr.uconn.edu.*
***Tel: 860-486-2382; e-mail: olgac@engr.uconn.edu.*

Abstract: Delayed Resonators (DR) refer to a special type of active vibration absorber system. An intentionally delayed feedback control is used to bring an absorber sub-structure to resonance, in turn canceling vibrations in the primary structure. So far, this theory has been applied to traditional mechanical vibration absorbers with proof masses. This study proposes a new venue for DR-based vibration control using piezoelectric networks. The inherent electro-mechanical coupling of piezoelectric materials has enabled the use of electrical circuits to absorb vibrations in mechanical structures. The DR principles are applied here to create a resonating electrical circuit which offers ideal vibration absorption performance. Experimental studies are also presented to demonstrate the results.

Keywords: Time-delay, vibration control, delayed resonator, piezoelectric materials

1. INTRODUCTION

Delayed Resonator (DR) theory was introduced in Olgac & Holm-Hansen (1994) as they proposed to utilize time-delayed controls on an ordinary active vibration absorber in order to impart resonance. It is known that resonant substructures offer ideal vibration absorption performance (Inman, 2014). Interestingly, however, control theorists usually regard time-delays as contaminants that may induce instabilities in feedback-controlled systems (Richard, 2003). The DR philosophy carefully feeds upon these two facts and proposes an alternative method for improved vibration control. An active element (e.g., actuator) is added to the absorber, which exerts force (or torque) according to a feedback law. If the feedback signal is intentionally delayed, it might be possible to induce marginal stability (i.e., resonance) in the absorber substructure at a certain frequency. Most naturally, a critical argument following this proposition is that the feedback parameters need to be tuned in-situ, such that the desired resonance and ultimate vibration suppression can be achieved. Moreover, the combined system consisting of the primary and absorber structures becomes a time-delayed system. These are infinite-dimensional systems and their characteristics are governed by an infinite spectrum. Thus they offer many design challenges for control synthesis, such as ensuring asymptotic stability.

Over the years, research and ensuing publications on DR theory addressed the issues raised above. It was shown that with proportional controllers, the feedback gain and delay can be synthesized analytically, while taking the combined system's stability into account (Olgac & Holm-Hansen, 1995). The concept was demonstrated in several experiments, including distributed systems as primary structures (Olgac et al., 1997). Centrifugal delayed resonators were also developed for torsional vibration problems (Hosek et al., 1997; Filipovic & Olgac, 2002).

Along a different vein of research, piezoelectric materials attracted popularity for vibration control purposes in the 80s (Forward, 1979; Bailey & Ubbard, 1985). These materials possess a unique characteristic by exhibiting stress/strain in response to an applied electrical field. Conversely, if the material is mechanically subjected to stress/strain, it generates an electrical field across its electrodes. This inherent electro-mechanical coupling can be used for sensing, actuation and control purposes. Hagood & von Flotow (1991) proposed the idea of implementing passive shunt circuits for structural vibration damping. The piezoelectric element, which acts as a capacitor in an electrical circuit, was connected to a resistor (R) and inductor (L), all in series. They demonstrated that through the proper tuning of the R-L parameters, significant vibration damping could be achieved. This proposition influenced further research and many different circuit configuration including active voltage sources were considered. Wu (1996) studied a variation where the R-L elements were connected in parallel, and demonstrated that similar vibration control performance could be achieved. Tang & Wang (2001) offers an in-depth review on various piezoelectric network setups.

In this work we pursue a new direction of application for DR theory. We consider an active piezoelectric network in lieu of a mechanical vibration absorber, as the DR implementation platform. A series R-L circuit is to be used with an added voltage source to exert the feedback control. This circuit is shunted to a piezoelectric patch, which is in turn bonded on a cantilever beam. The beam serves as the primary structure subject to external vibration. After a brief review on DR theory, we present the modeling approach for the coupled beam and circuit system. We propose a practicable delayed feedback control strategy in the circuit. Applying principles of DR theory, we show that the circuit can be made resonant at desired frequencies to absorb the beam's vibration. The derived theory is demonstrated on an experimental platform. Practical issues and design considerations are also discussed.

10.1016/j.ifacol.2016.07.475

2. REVIEW OF DELAYED RESONATOR THEORY

A brief overview on delayed resonator theory is presented in this section. Consider a DR absorber as seen in Fig. 1 (Olgac et al., 1997). A delayed displacement feedback is applied through an actuator for the active tuning of the device.

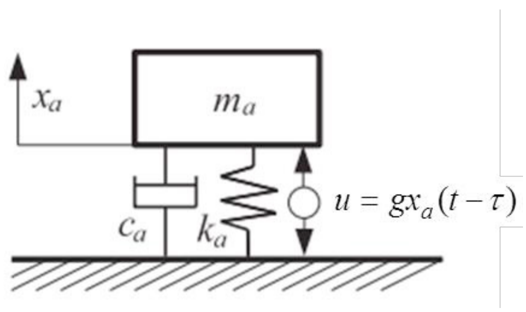

Fig. 1. Delayed resonator vibration absorber with acceleration feedback.

The governing equation for the absorber is

$$m_a\ddot{x}_a(t)+c_a\dot{x}_a(t)+k_a x_a(t)-gx_a(t-\tau)=0\,. \tag{1}$$

In Laplace domain, the characteristic equation becomes

$$m_a s^2+c_a s+k_a-g\,\mathrm{e}^{-\tau s}=0 \tag{2}$$

Notice that because of the added feedback delay (2) is a quasi-polynomial and has infinitely many roots. For a desired DR operation, the feedback gain g and the delay τ are selected such that the characteristic equation exhibits a pair of roots at $\pm\omega\mathrm{i}$. By substituting $s=\omega\mathrm{i}$ in (2), one can solve the necessary control parameters as

$$g(\omega)=(-1)^{l-1}\sqrt{(k_a-m_a\omega^2)^2+(c_a\omega)^2}\;,$$
$$\tau(\omega)=\frac{1}{\omega}\left[\tan^{-1}\left(\frac{c_a\omega}{m_a\omega^2-k_a}\right)+(l-1)\pi\right],\; l=1,2,\ldots. \tag{3}$$

Here, the counter l is called the *branch number*. Due to the periodicity of $\mathrm{e}^{-\tau\omega\mathrm{i}}$, for a given ω, the delay parameter repeats indefinitely on so-called *branches*. When the control parameters are tuned according to (3), the absorber exhibits a *resonant* mode at frequency ω. As such, it could absorb all vibrations at ω from a primary structure it is attached to. Typically the gain and delay parameters are plotted in loci, as shown in Fig. 2 for illustrative purposes. This figure represents a so-called *stability map* for the absorber substructure. The shaded zone corresponds to parameter compositions for which the absorber is asymptotically stable.

When the DR is mounted on a primary structure, the coupled dynamics would be governed by a different characteristic equation. The asymptotic stability of this combined system needs to be taken into account so that the DR tuning does not cause instability in the combined system dynamics. More details on such considerations are available in DR literature such as Olgac et al. (1997) and Filipovic & Olgac (2002). These concepts are also revisited later in this text.

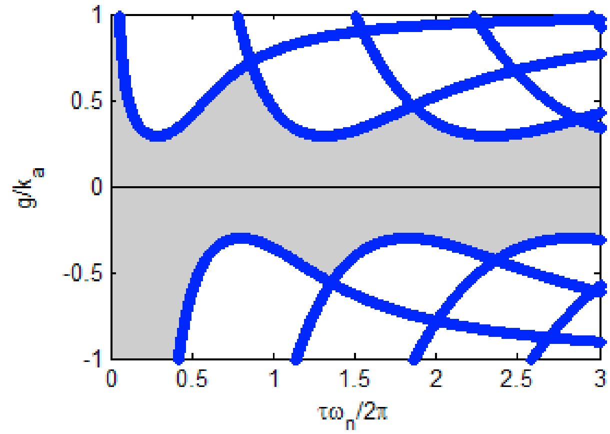

Fig. 2. Normalized delay and gain parameters for delayed resonator tuning (m_a = 1 kg, c_a = 12 Ns/m, k_a = 1600 N/m). Stable region shaded.

3. DELAYED RESONATOR APPLICATION IN PIEZOELECTRIC NETWORKS

3.1 Transverse vibrations in a cantilever beam coupled to a piezoelectric shunt circuit

For the conceptual work, a generic cantilever beam is used as the host (primary) structure. Piezoelectric patches are bonded to either sides of the beam. One patch is used to generate external excitation, while the other is connected to a shunt circuit for control purposes. Illustrated in Fig. 3, this configuration is a typical benchmark setup (Hagood & von Flotow, 1991; Tang & Wang, 2001).

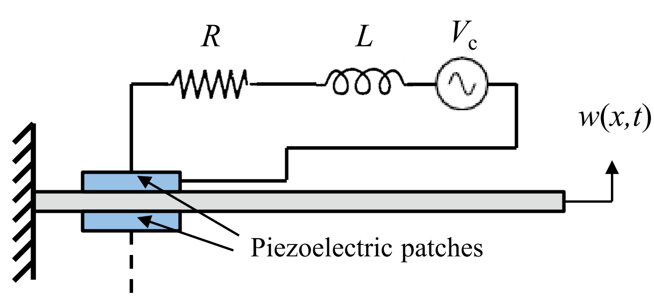

Fig. 3. Cantilever beam with bonded piezoelectric patches and shunt circuit.

In this work, the beam and circuit dynamics are analyzed following an approach adopted from Tang & Wang (2001). The equations of motion for the system are derived starting from an assumed mode truncation. The transverse vibration along the beam's longitude is taken as

$$w(x,t)=\phi_1(x)u(t) \tag{4}$$

where ϕ_1 is the shape function corresponding to the beam's first bending mode and u denotes temporal dynamics. The first mode assumption and piezoelectric constitutive relations are used in conjunction with Hamilton's principle to derive the following equations of motion.