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ROBUST CONTROLLER DESIGN FOR NEUTRAL SYSTEMS WITH DISTRIBUTED TIME-DELAY ⋆

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Abstract: Robust stabilizing controller design for neutral time-delay systems with distributed time-delay is considered. The uncertainties in the system are bounded by using a frequency-dependent function. This bound is then used in the proposed design approach. Finally, an example is presented to demonstrate this approach.

Keywords: Time-delay systems; neutral systems; robust control; controller design; stabilization.

1. INTRODUCTION

There are many examples of time-delay systems where the delays affect the system in a distributed manner (e.g., see Xie et al. (2001)). Furthermore, in some of these systems, the derivative of the state vector may also be subject to distributed time-delays or some part of the dynamics may be described by distributed-delay-algebraic equations coupled with distributed-delay-differential equations. In both of these cases, the system in question becomes a neutral system as opposed to a retarded system, where the dynamics are described by purely delay-differential equations where the derivative of the state vector is not subject to any time-delays (Niculescu (2001)). Furthermore, as any physical system, such systems may not be modelled exactly (Santos et al. (2006)). Therefore, robust controller design approaches are important for such systems (Zhong (2006)).

In the present work, we propose a robust stabilizing controller design approach for neutral systems with distributed time-delay. The proposed design approach uses a frequency-dependent robustness bound. Such a bound was first defined for delay-free systems by İftar and Özgüner (1987a,b) and for retarded time-delay systems by İftar (2008, 2014). Neutral systems were considered by İftar (2015), where only discrete time-delays were considered. In here, however, we consider systems which may be subject to distributed time-delays besides discrete time-delays. The problem is formally defined in Section 2 and the proposed approach is presented in Section 3. An example is presented in Section 4 to demonstrate the proposed approach. Finally, concluding remarks are given in Section 5.

Throughout the paper, $\mathbf{R}$ and $\mathbf{C}$ denote the sets of, respectively, real and complex numbers. I denotes the identity matrix of appropriate dimensions. For $s \in \mathbf{C}$, $\mathrm{Re}(s)$ and $|s|$ denote the real part of s and the magnitude of s, respectively. For a positive integer k, $\mathbf{R}^k$ denotes the k-dimensional real vector space. For a vector function $x(\cdot)$, $\dot{x}(t)$ is the derivative of $x(t)$ with respect to t. $\bar{\sigma}(\cdot)$, $\underline{\sigma}(\cdot)$, $\det(\cdot)$, and $\mathrm{rank}(\cdot)$ respectively denote the maximum singular value, the minimum singular value, the determinant, and the rank of the indicated matrix. Finally, $j := \sqrt{-1}$ is the imaginary unit.

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2. PROBLEM STATEMENT

Consider a linear time-invariant (LTI) neutral system with distributed time-delay, whose dynamics are described as:

$$\dot{x}(t) + \int_{-\tau}^{0} E(\theta)\dot{x}(t+\theta)d\theta = \int_{-\tau}^{0} \left(A(\theta)x(t+\theta) + B(\theta)u(t+\theta)\right) d\theta \qquad (1)$$

$$y(t) = \int_{-\tau}^{0} C(\theta)x(t+\theta)d\theta \qquad (2)$$

where, $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^p$, and $y(t) \in \mathbf{R}^q$ are, respectively, the state, the input, and the output vectors at time t and $\tau > 0$ is the maximum time-delay in the system. Furthermore, $E(\theta)$, $A(\theta)$, $B(\theta)$, and $C(\theta)$ are matrix functions defined for $-\tau \le \theta \le 0$, which are assumed to be bounded, except at finitely many points at which they may contain a Dirac delta term. $E(\theta)$, however, is assumed not to contain a Dirac delta term at $\theta = 0$ (i.e., $E(0)$ is assumed to be bounded) but may contain terms $\delta(\theta+h)$ for $0 < h \le \tau$, where $\delta(\cdot)$ indicates the Dirac delta function, which is defined as

$$\int_{-\tau}^{0} \delta(\theta+h)\xi(t+\theta)d\theta = \xi(t-h) \ , \qquad (3)$$

for any continuous function $\xi(\cdot)$ and $0 \le h \le \tau$. The inclusion of Dirac delta terms in these matrix functions allow the representation of discrete time-delays, together with distributed time-delays. Here, it is assumed that the matrix functions $E(\theta)$, $A(\theta)$, and $B(\theta)$ may be subject to

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uncertainties. Precisely, it is assumed that these matrix functions can be written as $E(\theta) := E^n(\theta) + E^u(\theta)$, $A(\theta) := A^n(\theta) + A^u(\theta)$, and $B(\theta) := B^n(\theta) + B^u(\theta)$, where the matrix functions with superscript n are the *nominal* and those with superscript u are the *uncertain* parts. The nominal parts are assumed to be known matrix functions. The uncertain parts, however, are not known but are assumed to satisfy

$$\int_{-\tau}^{0} \bar{\sigma}\left(E^u(\theta)\right) d\theta \le \epsilon \,, \qquad \int_{-\tau}^{0} \bar{\sigma}\left(A^u(\theta)\right) d\theta \le \alpha \,, \tag{4}$$

and

$$\int_{-\tau}^{0} \bar{\sigma}\left(B^u(\theta)\right) d\theta \le \beta \,, \tag{5}$$

where ϵ, α, and β are known nonnegative bounds. Since the input-output uncertainties in a system can be represented either at the input or at the output, we assume that the output equation (2) is free of any uncertainties; i.e., $C(\theta)$ is a known matrix function.

It is known that the system (1) is stable if and only if it does not have any modes with real part greater than or equal to $-\rho$ for some $\rho > 0$ (Hale and Verduyn-Lunel (1993)). Here $s_o \in \mathbf{C}$ is said to be a *mode* of the system (1) if $\det\left(s_o\bar{E}(s_o) - \bar{A}(s_o)\right) = 0$, where

$$\bar{E}(s) := I + \int_{-\tau}^{0} E(\theta)e^{s\theta}d\theta \ \text{ and } \ \bar{A}(s) := \int_{-\tau}^{0} A(\theta)e^{s\theta}d\theta \,. \tag{6}$$

Furthermore, we call a mode as an *unstable mode* if it has a nonnegative real part. Although the system (1) may have infinitely many modes, it is known that, for any $\rho > 0$, it has only finitely many modes with real part greater than or equal to $\mu + \rho$, where

$$\mu := \sup\left\{\mathrm{Re}(s) \mid \det\left(\bar{E}(s)\right) = 0\right\} \,, \tag{7}$$

where $\bar{E}(s)$ is as defined in (6) (Hale and Verduyn-Lunel (1993)).

The *nominal model* for the *actual system* (1)–(2) is described as:

$$\dot{x}(t) + \int_{-\tau}^{0} E^n(\theta)\dot{x}(t+\theta)d\theta = \int_{-\tau}^{0} \left(A^n(\theta)x(t+\theta) + B^n(\theta)u(t+\theta)\right) d\theta \tag{8}$$

$$y(t) = \int_{-\tau}^{0} C(\theta)x(t+\theta)d\theta \tag{9}$$

The problem, then, is to design a controller based on the nominal model (8)–(9) such that, when this controller is applied to the actual system (1)–(2), the closed-loop system is robustly stable for all uncertainties satisfying the bounds (4)–(5). Before presenting our proposed solution to this problem, however, we present the following two assumptions.

Assumption 1: For any $E^u(\theta)$, satisfying (4), $\mu < 0$, where μ is as defined in (7).

Assumption 2: For any $E^u(\theta)$ and $A^u(\theta)$, satisfying (4), the number of unstable modes of the system (1) is the same.

Note that, Assumption 1 implies that the number of unstable modes of the system (1) is finite for any uncertainties satisfying (4)–(5). This is, in fact, a necessary condition for the stabilizability of the system (1)–(2) by a proper controller (Loiseau et al. (2002)). Once this assumption is satisfied, Assumption 2 implies that the number of unstable modes of the actual system (1)–(2) and of the nominal system (8)–(9) are the same for any uncertainties satisfying (4)–(5).

3. PROPOSED DESIGN APPROACH

The transfer function matrix (TFM) of the actual system (1)–(2) is given as

$$G(s) = \bar{C}(s)\left(s\bar{E}(s) - \bar{A}(s)\right)^{-1}\bar{B}(s) \tag{10}$$

where $\bar{E}(s)$ and $\bar{A}(s)$ are as defined in (6),

$$\bar{B}(s) := \int_{-\tau}^{0} B(\theta)e^{s\theta}d\theta \ \text{ and } \ \bar{C}(s) := \int_{-\tau}^{0} C(\theta)e^{s\theta}d\theta \,. \tag{11}$$

On the other hand, the TFM of the nominal system (8)–(9), is given as

$$G^n(s) = \bar{C}(s)\left(s\bar{E}^n(s) - \bar{A}^n(s)\right)^{-1}\bar{B}^n(s) \tag{12}$$

where $\bar{E}^n(s)$, $\bar{A}^n(s)$, and $\bar{B}^n(s)$ are defined similar to $\bar{E}(s)$, $\bar{A}(s)$, and $\bar{B}(s)$ with $E(\theta)$, $A(\theta)$, and $B(\theta)$ respectively replaced by $E^n(\theta)$, $A^n(\theta)$, and $B^n(\theta)$.

Now, let $\Gamma(s)$ be such that

$$G(s) = G^n(s)\left(I + \Gamma(s)\right) \,. \tag{13}$$

We can then obtain a frequency-dependent upper bound on the norm of $\Gamma(j\omega)$ as follows.

Lemma 1: Suppose that (4)–(5) hold. Let

$$\gamma(\omega) := \frac{\gamma_n(\omega)}{\gamma_d(\omega)} \,, \tag{14}$$

where

$$\gamma_n(\omega) := \beta + \left(|\omega|\epsilon + \alpha\right)\bar{\sigma}\left(G_o(j\omega)\right) \tag{15}$$

and

$$\gamma_d(\omega) := \underline{\sigma}\left(\bar{B}^n(j\omega)\right) - \left(|\omega|\epsilon + \alpha\right)\bar{\sigma}\left(G_o(j\omega)\right) \,, \tag{16}$$

where

$$G_o(s) := \left(s\bar{E}^n(s) - \bar{A}^n(s)\right)^{-1}\bar{B}^n(s) \,. \tag{17}$$

Suppose that $\gamma_d(\omega) > 0$, $\forall\omega \in \mathbf{R}$. Then,

$$\bar{\sigma}(\Gamma(j\omega)) \le \gamma(\omega) \,, \qquad \forall\omega \in \mathbf{R} \,. \tag{18}$$