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Multi-criteria optimisation design of shapers with piece-wise equally distributed time-delay

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Abstract: The paper presents an optimisation based method to parametrize input shapers with time delays of piece-wise-equal distribution. The design respects the two key requirements on the shaper performance that are the prompt response time and the robustness represented by residual vibration functions, which are however conflicting criteria as a rule. Thus, the optimisation task is solved as a constrained multi-objective problem where the constraints respect the additional requirements on the shaper performance such as a non-decreasing step response or zero residual function at some frequency points. The result is a Pareto front that shows a trade-off between the two main requirements. Next to the presentation of the results in an example, a user friendly shaper-design tool is presented.

Keywords: time-delay, signal shapers, vibration suppression, multi-objective optimisation, Pareto frontier

1. INTRODUCTION

The idea of shaping reference commands in order to remove oscillatory modes from the signal was published by Smith (1957), where feed-forward input shaping based on the time delays was introduced as so-called PosiCast, known also as the zero-vibration (ZV) shaper. Advantageous robust shapers as zero-vibration-derivative (DZV) or extra-insensitive (EI) were introduced and investigated by Singer and Seering (1990), Singhose et al. (1994). Further modifications followed. Multi-mode shapers were developed to compensate two or more flexible modes, see Singh and Heppler (1993), Singh and Vadali (1995) or Sung and Singhose (2009). Next to the continuous-time domain shapers, discrete versions of the shaper were firstly published by Murphy and Watanabe (1992), followed by discussion about implementation aspects by Magee and Book (1993) and more recent publications by Baumgart and Pao (2007), Cole (2011). A comparison and an comprehensive analysis of robustness of signal shaping techniques were published by Singer and Seering (1990) or more recently by Vaughan et al. (2008).

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As an alternative to signal shaping, various command profiles, such as trapezoidal, S-curve or trigonometric functions can be used to smooth fast changes in the reference input signal of flexible systems, Meckl (1998), Singhose et al. (2010). As shown in Vyhlídal and Hromčík (2015), these finite-length input filters can be represented by distributed time delays. This applies also for the Jerk-limited input shapers proposed by Singh (2004). Compared to the classical shapers, the output of the signal is smoothed by various types of filters to avoid strong discontinuities after shaping the signals. In Vyhlídal et al. (2012), Vyhlídal et al. (2013b), an alternative approach was proposed, by considering an equally distributed delay directly in the shaper design. Subsequently, more complex delay distributions were considered in Vyhlídal et al. (2013c). Next to the smoothening effect at the signal accommodation part, the retarded characteristics of the shaper spectrum can be considered as an implementation benefit, particularly, if the shaper is implemented within a closed loop system Vyhlídal et al. (2013a), Vyhlidal et al. (2015).

This paper is a subsequent work to Vyhlídal and Hromčík (2015), where an optimisation based method was proposed for the direct design of robust shapers with piece-wise equally distributed delays. The routine was based on constrained linear least squares optimisation applied directly to the minimization of the sensitivity function in the frequency domain. The key contribution with respect to Vyhlídal and Hromčík (2015) is in considering not only the

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robustness, but also the action time in the shaper design. Next, the design task is reformulated a constrained multi-objective optimization problem, which can be solved using convex optimization techniques. Besides, a software design tool is presented. The paper is also related to the recent work Pilbauer et al. (2015), where an alternative approach was applied to a shaper with continuous distribution of the delay by optimising smooth kernel functions.

2. PRELIMINARIES

The objective of applying input shapers in a serial interconnection with a system is to fully or partially compensate its oscillatory mode, which is determined by a couple of poles $\hat{s}_z = -\beta \pm j\Omega, \beta = \omega\zeta, \Omega = \omega\sqrt{1-\zeta^2}$, where ζ, ω are the damping ratio and natural frequency of the mode to be compensated. As mentioned in the previous section, this task can be performed by a number of shaper classes. A common property of the above mentioned shapers is the neutral spectrum of shaper zeros, as demonstrated in Vyhlídal et al. (2013a), Vyhlidal et al. (2015). Thus, next to the *active* shaper zeros that compensate the effect of the oscillatory pole, there are infinitely many higher frequency zeros distributed close to the imaginary axis or even in the right half plane. Once, the shaper is included in the feedback loop as an inverse shaper, these zeros can be projected to the closed loop poles and can be responsible for the stability loss. For example, this will happen if the scheme in Fig. 1 is applied to handle the task of oscillatory mode suppression of System 2 when induced not only by the set-point w, but also by disturbance $d_{1,2}$, see Vyhlídal et al. (2013a), Vyhlidal et al. (2015). The only option for the effective loop scheme in Fig. 1 is to involve a shaper with a distributed time delay, as its spectrum of zeros is retarded - i.e. the high frequency zeroes lie safely far from the stability boundary Vyhlidal et al. (2015); Vyhlídal et al. (2013b); Vyhlídal and Hromčík (2015).

2.1 Input shapers with distributed delay

A general form of a shaper with distributed delay can be described as

$$u(t) = Aw(t) + (1-A)\int_0^T w(t-\eta)dh(\eta). \qquad (1)$$

with the parameter $A \in [0,1]$ and the delay distribution $h(\eta)$ over the finite length segment $\eta \in [0,T]$, satisfying $h(\eta) = 1, \eta \geq T$. A common requirement on the delay distribution is the non-decreasing step response shape over $\eta \in [0,T]$. The transfer function of the distributed delay (1) is given by

$$G(s) = A + (1-A)F(s,T), \qquad (2)$$

where $F(s,T) = L\left\{\int_0^T w(t-\eta)dh(\eta)\right\}$ is the transfer function of the delay. The spectral synthesis of the shaper (1) consists of placing the dominant zero of (2) to the position of the oscillatory pole $\hat{s}_z$. As already mentioned, the rest of the infinitely many zeros of (2) follows the exponential asymptotic curves departing of the stability boundary with increasing moduli of the roots.

In Vyhlídal et al. (2012), Vyhlídal et al. (2013b), a zero vibration shaper with equally distributed delay (DZV) was proposed, considering $F(s) = \frac{1-e^{-sT}}{sT}$. Next to the retarded spectrum, a smoother accommodation and filtering effect are the benefits compared to the classical ZV shaper with the distributed delay. A negative aspect compared to the classical ZV shaper was the increased response time length. This inefficiency can partly be overcome by combining the lumped and equally distributed delay $F(s,T,\alpha) = \frac{e^{-\alpha Ts}-e^{-Ts}}{s(1-\alpha)T}$, see Vyhlídal et al. (2013c), where parameter α determines the ratio between the lengths of the lumped part and the overall delay length T. An extension towards more general delay distributions, e.g. with S-shape $F(s) = \frac{4(1-2e^{-s\frac{T}{2}}+e^{-sT})}{T^2s^2}$ or trigonometric delay $F(s,T) = \frac{4\pi^2\left(1-e^{-Ts}\right)}{Ts(T^2s^2+4\pi^2)}$ was proposed in Vyhlídal and Hromčík (2015), along with a fully analytical parameterization procedure.

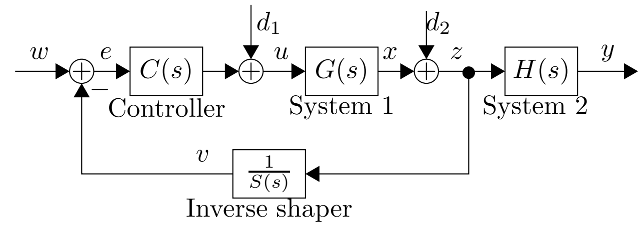

Fig. 1. Inverse shaper in the feedback

In order to increase robustness, i.e. provide distributed delay alternatives to the classical EI, DZV shapers, a least squares approach was proposed in Vyhlídal and Hromčík (2015), considering

$$F(s) = \frac{\sum_{k=0}^{N} \bar{a}_k e^{-s\tau_k}}{s}. \qquad (3)$$

In the procedure, the delays $\tau_k, k = 0..N$ covering equidistantly the interval the delay length interval $[0,T]$ are fixed and the parameters A and $\bar{a}_k, k = 0..N$ are free parameters to optimise a residual vibration characteristic.

A technique using smooth kernel function was proposed by Pilbauer et al. (2015) where transfer function of the shaper is given by

$$G(s) = A + \int_0^T g(\theta)e^{-s\theta}d\theta. \qquad (4)$$

with the kernel function g chosen as the polynomial $g(\theta) = \sum_{i=0}^{N_p} a_i\theta^i$. This approach has more degrees of freedom in the design but leads to more complicated multi-objective optimisation problem. Compared to (2)-(3), the resulting shaper is also of an enhanced complexity from the implementation point of view.

2.2 Residual vibration function

The idea of measuring robustness of the shaper was introduced by Singhose et al. (1994), where term *residual vibrations* were firstly used. The residual vibration function takes into account changes of the parameters of System 2 (Fig. 1). The robustness in this paper is expressed in terms of residual vibrations and more precisely in terms of the transfer function

$$V(\zeta,\omega) = \left|G\left(-\omega\zeta - j\omega\sqrt{1-\zeta^2}\right)\right| e^{\zeta\omega T}, \qquad (5)$$

which expresses the amplitude of the residual vibrations at the time $t = T$. The transfer function has uncertainties