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IFAC-PapersOnLine 49-10 (2016) 118–123

Obtention of the functional of complete type for neutral type delay systems via a new Cauchy formula ⋆

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Abstract: In this paper, we present a new manner for computing the so-called new form of the functional of complete type for neutral type time-delay systems without assumptions of continuity and differentiability on the initial function. It is obtained by using a new Cauchy formula. We present briefly the necessary stability conditions depending on the delay Lyapunov matrix that this result makes possible to prove.

Keywords: Neutral type delay systems; Lyapunov-Krasvoskii functionals; functionals of complete type; delay Lyapunov matrix.

1. INTRODUCTION

The study of linear time delay systems in the framework of Lyapunov-Krasovskii functionals with prescribed derivative was initiated by Repin (1965), and its first use for studying neutral type delay systems remount to the work of Castelan and Infante (1979). A significant formalization of the approach and substantial advances can be found in the contributions Rodriguez et al. (2004), Kharitonov (2005), Kharitonov (2013). In particular, the central role of the delay Lyapunov matrix is clarified, and the addition of a new term to the functional (which is then called of complete type) allows showing it admits a quadratic lower bound. This last property leads to robust stability bounds, Rodriguez et al. (2004), and exponential estimates of the system response Kharitonov (2005). Other results are the determination of critical frequencies and/or parameters Ochoa et al. (2013), and the extension of the predictor control scheme for neutral type systems with state and input delay Kharitonov (2015).

The procedure for constructing an explicit expression of the functional under the assumption that the system is exponentially stable, consists basically of two steps: (i) the prescription of a negative quadratic time derivative of the functional; (ii) its integration and the substitution of the Cauchy formula. In the early results in Bellman and Cooke (1963), assumption of differentiability of initial functions leads to a Cauchy formula depending on the derivative of the initial function. The integration by parts of the functional introduced in Rodriguez et al. (2004) allowed to present in Kharitonov (2005), under the same assumptions for the initial conditions, a new expression of the functional that avoids the derivative of the initial function, but contains the first and the second derivative of the delay Lyapunov matrix.

In this contribution, we present a new Cauchy formula that requires neither the differentiability, nor even the continuity of the initial function, and by the above described procedure, we arrive at the expression of the functional obtained in Kharitonov (2005). This subtlety is crucial for extending the necessary stability conditions depending on the delay Lyapunov matrix obtained for the pointwise delay case in Egorov and Mondié (2014), to the case of neutral type delay systems presented in Gomez et al. (2015), because an important element of the proof consists in using a particular class of initial functions that depends on the neutral type delay system fundamental matrix, which is not continuous at values multiples of the delay.

The paper is organized as follows. In Section 2, we provide basic facts of the fundamental and Lyapunov matrices for neutral type delay systems. The new Cauchy formula is proven in Section 3 and it is shown that the Cauchy formula introduced in Bellman and Cooke (1963) is a particular case of the new one. In Section 4, we use the Cauchy formula for computing the functional. Finally, in Section 5, we present without proof and illustrate the necessary stability conditions that motivate the development of the results, and we conclude with some comments.

Notation: The Euclidian norm for vectors is represented by $\|\cdot\|$. *The set* $\mathcal{O}(\tau)$ *is defined as* $\mathcal{O}(\tau) = [0,\infty)\backslash\{\tau+ih\}_{i=0}^{\infty}$. *For the derivative of the function* $F(\cdot)$ *with respect to the unique argument we use* $F'(\cdot)$. *The notation a.e. means "almost everywhere".* $\{A_{ij}\}_{i,j=1}^{r}$ *denotes a square matrix, where* A_{ij} $(i,j=1,...,r)$ *is the element in the i-th row and the j-th column, and* $Q>0$ *denotes a positive definite matrix. The space of* $\mathbb{R}^n$*-valued piecewise continuous functions on* $[-h,0]$ *is denoted by* $PC([-h,0],\mathbb{R}^n)$ *and the right hand-side (left hand-side) limit* $\lim_{\varepsilon\to 0} f(t+|\varepsilon|)$ $(\lim_{\varepsilon\to 0} f(t-|\varepsilon|))$ *is represented by* $f(t+0)$ $(f(t-0))$.

⋆ Project CONACYT 180725.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.07.495

2. PRELIMINARIES

Let us consider a neutral type time-delay system

$$\frac{d}{dt}\left(x(t) - Dx(t-h)\right) = A_0 x(t) + A_1 x(t-h), \ \ t \geq 0, \quad (1)$$

where $h > 0$ is the delay and D, A_0 and A_1 are constant matrices in $\mathbb{R}^{n\times n}$. The initial function $\varphi(\theta)$, $\theta \in [-h, 0]$, belongs to the space $PC^{(1)}([-h,0],\mathbb{R}^n)$, i.e.,

(1) $\varphi \in PC([-h,0],\mathbb{R}^n)$,
(2) the derivative $\varphi' \in C\left([-h,0]\setminus S, \mathbb{R}^n\right)$, where S is a finite set,
(3) there exists a number $\bar{\varphi}$ such that $\|\varphi'(\theta)\| \leq \bar{\varphi}$ for every $\theta \in [-h,0]\setminus S$.

The restriction of the solution $x(t,\varphi)$ to the interval $[t-h,t]$ is denoted by

$$x_t(\varphi) : \theta \to x(t+\theta,\varphi), \ \ \theta \in [-h,0].$$

From now on, we consider that the following assumptions hold:

(1) The solution $x(t,\varphi)$ satisfies system (1) almost everywhere.
(2) The function $x(t,\varphi) - Dx(t-h,\varphi)$ is continuous on $[0,\infty)$ and differentiable for $t \geq 0$ almost everywhere, and the right-hand side derivative is assumed to exist at $t = 0$.

Definition 1. (Bellman and Cooke (1963)). System (1) is said to be exponentially stable, if every solution of the system satisfies the inequality

$$\|x(t,\varphi)\| \leq \gamma e^{-\sigma t} \sup_{\theta\in[-h,0]} \|\varphi(\theta)\|, \ t \geq 0,$$

for $\sigma > 0$ and $\gamma \geq 1$.

The fundamental matrix $K(t)$ of system (1) satisfies the equation (see Bellman and Cooke (1963)) for $t \geq 0$ almost everywhere:

$$\frac{d}{dt}\left(K(t) - K(t-h)D\right) = K(t)A_0 + K(t-h)A_1, \quad (2)$$

with the initial conditions $K(\theta) = 0$ for $\theta \in [-h, 0)$, $K(0) = I$, and the sewing condition:

$K(t) - K(t-h)D$ is right-hand side continuous for $t \geq 0$.

From the Laplace transform of equation (2) it is easy to see that the fundamental matrix is also a solution of the equation

$$\frac{d}{dt}\left(K(t) - DK(t-h)\right) = A_0 K(t) + A_1 K(t-h).$$

According to the definition, the fundamental matrix is discontinuous at points νh, $\nu = 0, 1, 2, \ldots$. The jumps are described in the following lemma .

Lemma 1. (Bellman and Cooke (1963)). The fundamental matrix $K(t)$ has jumps at points νh, $\nu = 0, 1, 2, \ldots$ and their size values are determined by

$$\Delta K(\nu h) = D^{\nu},$$

where $\Delta K(\nu h) = K(\nu h + 0) - K(\nu h - 0)$.

If system (1) is exponentially stable, the matrix function

$$U(\tau) = \int_0^{\infty} K^T(t) W K(t+\tau) dt \quad (3)$$

is defined as the delay Lyapunov matrix associated with matrix W(see Rodriguez et al. (2004), Kharitonov (2005)). The Lyapunov matrix is continuous for $\tau \in \mathbb{R}$ (Lemma 6.3 in Kharitonov (2013)), continuously differentiable at $\xi \neq jh$, $j = 0, \pm 1, \ldots$, and satisfies the following properties:

(1) Dynamic

$$\frac{d}{d\tau}\left(U(\tau) - U(\tau - h)D\right) = U(\tau)A_0 + U(\tau-h)A_1, \ \ \tau \geq 0, \ a.e., \quad (4)$$

(2) Symmetry

$$U^T(\tau) = U(-\tau), \ \ \tau \geq 0, \quad (5)$$

(3) Algebraic

$$\begin{aligned} A_0^T U(0) + U(0)A_0 + A_1^T U(h) + U(-h)A_1 \\ - \left(A_0^T U(-h) + A_1^T U(0)\right) D \\ - D^T \left(U(h)A_0 + U(0)A_1\right) = -W. \end{aligned} \quad (6)$$

In Kharitonov (2013), it is shown that the algebraic property can also be written as

$$-W = \Delta U'(0) - D^T \Delta U'(0) D,$$

where $\Delta U'(0) = U'(+0) - U'(-0)$.

The jumps of the first derivative of the Lyapunov matrix are characterized in the next lemma.

Lemma 2. (Kharitonov (2005)). The jump size values of the first derivative of the Lyapunov matrix $U(\tau)$ at points $j = 0, 1, 2, \ldots$, are given by

$$\Delta U'(jh) = \Delta U'(0) D^j,$$

where $\Delta U'(jh) = U'(jh + 0) - U'(jh - 0)$.

Remark 1. Due to the discontinuities of the fundamental matrix $K(t+\tau)$ at $t_j = jh - \tau$, $j = 0, 1, \ldots$,

$$\begin{aligned} U'(\tau) = \int_{\mathcal{O}(-\tau)} K^T(t) W \frac{d}{d\tau} K(t+\tau) dt \\ + \sum_{j=0}^{\infty} K^T(jh-\tau) W \Delta K(jh), \ \ \tau \in (-h,0) \cap (0,h). \end{aligned} \quad (7)$$

3. A NEW CAUCHY FORMULA

In this section, we present a new Cauchy formula of system (1) given in terms of the fundamental matrix. It is shown that this formula is a generalization of the one presented in Bellman and Cooke (1963).

Lemma 3. Given an initial function φ, the solution $x(t,\varphi)$ of system (1) is determined by

$$\begin{aligned} x(t,\varphi) = & K(t)\left(\varphi(0) - D\varphi(-h)\right) \\ & + \int_{-h}^{0} K(t-h-\theta) A_1 \varphi(\theta) d\theta \\ & + \frac{d}{dt}\left(\int_{-h}^{0} K(t-\theta-h) D\varphi(\theta) d\theta\right), \ t \geq 0. \end{aligned} \quad (8)$$

Proof. Integrating system (1) we get the expression

$$\begin{aligned} x(t) = & Dx(t-h) + \varphi(0) - D\varphi(-h) + A_0 \int_0^t x(\theta) d\theta \\ & + A_1 \int_{-h}^{t-h} x(\theta) d\theta, \ t \geq 0. \end{aligned}$$

For $\xi \in [0,t)$, we consider the term

$$J(t,\xi) = \left(K(t-\xi) - K(t-\xi-h)D\right) \int_0^{\xi} x(\theta) d\theta.$$