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An Output Feedback Multivariable PD-Controller Design Method for Time Delay Systems: A New LMI Approach

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Abstract: This paper investigates the problem of designing a static output feedback controller and a PD multivariable static output feedback controller for linear time delay systems within the LMI framework. The main advantage of the proposed approach is its powerful ability to decouple the input and the output gain matrices leading to synthesize the controller gain matrix through linear matrix inequalities (LMI) technique. The synthesis conditions are thus formulated in LMI form which avoids the use of any iterative approach to resolve the feasibility problem. Moreover, the use of a PD multivariable type of static output feedback controller provides more information to the time delay control systems which results in achieving more improvement on the maximum admissible delay bound. From this point of view, it brings light on the expectation that if the controller is built up by as much information as possible, there exists a potential room for further improvement. In addition, it appears that the aim and thus the requirement of an LMI sort of formulation needs a reduced form of use of the free weighting matrices. Therefore, the challenge of an LMI formulation with the most generalistic attitude remains still open. Besides, the proposed methodology can be easily extended to time delay systems subject to time varying delay and/or model uncertainties, parameter perturbations and external disturbances. Finally, a dynamic output feedback can also be taken into consideration in a similar manner with the developed technique of the present work. A numerical example is presented to illustrate the application of the developed results. The numerical results indicate that some improvements on the maximum admissible delay bound are achieved with the proposed methodology in comparison to that reported in the literature.

Keywords: Dynamic systems, time delay, descriptor systems, Lyapunov stability, feedback stabilization, static output feedback, PD controller, numerical solutions.

1. INTRODUCTION

The delay effect is one of the significant phenomenon for dynamical systems. The state information concerning the physical plants are often subject to the past state that is the history of the state. Under such a circumstance, usually the stability and performance of the system can be heavily deteriorated. Both the stability and stabilization studies aim to investigate whether the system remains stable or if it can be stabilized regardless the delay effect. If this is not possible then the investigation pursues to find the largest admissible bound of the delay that the system tolerates and/or a stabilizing controller can be synthesized under the tolerated delay effect. A substantial amount of work has been conducted last several decades, see for example Gu *et al.* (2003) and Wu *et al*. (2010) and the references therein.

The output feedback control problem is one of the fruitful field of research within the community of control systems. There are two primary aspects of this strong and close interest among researchers. First, as the controller is based on the measurement of the output signal, this situation makes it more preferable in comparison to the state-feedback controller especially if the state information is not entirely available or expensive to estimate or sometimes even dangereous to access such as in a nuclear power plant. Second, the synthesis conditions of a static output feedback controller are usually obtained in the form of bilinear matrix inequalities (BMI) which are then usually solved via iterative based techniques. From this point of view, it involves a significant challenge of obtaining a feasible solution set for the existence of the static output feedback controller through the use of only LMI form of conditions.

The design of static output feedback controller concerning the time delay systems subject to constant delay or time varying delay is frequently studied in the literature, see for example (Tarn *et al.*, 1996; Wu and Chou, 1996; Chang *et* al., 2004; Park, 2005; Kharitonov *et al.*, 2005; Baser and Kizilsac, 2007; Suplin and Shaked, 2008; Yang and Ye, 2009; Yan *et al.*, 2010) and the references therein. Based on an LMI approach, dynamic output feedback H-infinity controller design problem is studied to stabilize nominal or uncertain time delay systems with the aim of satisfying a prescribed H-infinity performance level in (Jeung *et al.*, 1998; Azuma and Sagara, 2003; Su and Chu, 2007; Li and Jia, 2009). An observer-based output feedback controller is developed for a class of linear time delay systems subject to time-varying delay in (Shieh, 2002; Sun, 2002; Zhou *et al.*, 2013). Concerning an unknown input delayed system, the characterization of the static output feedback stabilization is

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achieved in terms of matrix inequalities in (Du *et al.*, 2009) which leads to a decoupling between input and the gain output matrix and thus avoids to impose any constraint on the Lyapunov matrix when the controller matrix is parametrized. The static and integral output feedback stabilization problems are addressed for state and/or input delayed continuous-time systems in (Du *et al.*, 2010). Utilizing the delayed output measurement, the design of a dynamic output feedback guaranteed cost controller is illustrated in (Thuan *et al.*, 2012) for a class of time delay systems subject to interval type of nonsmooth time varying delay. Moreover, the problem of static output feedback controller synthesis is investigated in (Liu *et al.*, 2015; Hao and Duan, 2015). It can be noticed that the control law proposed in (Du *et al.*, 2010) can be regarded as a proportional-integral (PI) multivariable dynamic output feedback controller. Hence, this gives a strong motivation for the present work to introduce a PD multivariable static output feedback controller for linear time delay systems. Another motivation is inspired from the challenge of being able to represent the conditions of the controller synthesis in terms of LMIs in order to allow the use of convex optimization techniques while searching a feasible solution set allowing a maximum admissible delay bound.

In this paper, the design problem of PD multivariable static output feedback controller is addressed for linear time delay systems. First of all, the research on this issue is conducted in two stages. In the first phase of the study, a pure static output feedback controller synthesis is developed while design of a PD multivariable static output feedback controller is illustrated in the subsequent stage. In order to succeed the goal of obtaining the static output feedback stabilization criteria through LMIs, the linear time delay system is first expressed in the form of descriptor systems representation (Fridman and Shaked, 2002). The control input term is maintained within the dynamics without any direct substitution from the static output feedback control law. By this approach, the decoupling of the input matrix and the output gain matrix is ensured. Moreover, the controller gain matrix and the Lyapunov-Krasovskii (L-K) matrices are also decoupled from each other potentially which overcomes any sort of constraint while choosing the L-K matrices. An augmented form of L-K functional is selected and some sufficient stabilization criteria are developed within the framework of LMIs. The controller synthesis conditions are relaxed by employing Wirtinger based inequality introduced in (Seuret and Gouaisbaut, 2013) and the free weighting matrices. A numerical example is given to show the effectiveness of the proposed methodology. The numerical results obtained for the maximum allowable delay bound exhibit significant progresses with respect to those existing in the literature.

2. PROBLEM STATEMENT

Let us consider a class of linear time delay systems defined in descriptor form (Fridman and Shaked, 2002) as follows

$$\begin{aligned} \dot{x}(t) &= z(t) \\ z(t) &= Ax(t) + A_d x(t-d) + Bu(t) \\ y(t) &= Cx(t) \\ x(t) &= \phi(t), \ t \in [-d, 0]. \end{aligned} \tag{1}$$

where $x(t) \in \Re^n$, $z(t) \in \Re^n$, $u(t) \in \Re^m$, $y(t) \in \Re^p$ represent the state vector of the system, the descriptor state, the exogeneous control input signal, the measured output signal, respectively, A, A_d, B, C denote constant system matrices with appropriate dimensions, $d > 0$ is an unknown constant delay term, $\phi(t)$ is a piecewise continuously differentiable function representing the initial conditions. We now consider a static output feedback controller of the following form

$$u(t) = Ky(t) \tag{2}$$

where $K \in \Re^{m \times p}$ represents the controller gain to be determined appropriately. The objective of the present work is to find the existence conditions for the static output feedback controller selected in (2) so that the closed-loop system (1) with (2) is guaranteed to be globally and asymptotically stabilized in the Lyapunov sense under a maximum admissible delay bound.

3. THE SYNTHESIS OF A STATIC OUTPUT FEEDBACK CONTROLLER

The conditions for the existence of a stabilizing static output feedback controller are summarized in the following lemma.

Lemma 1: If there exist real and symmetric (semi) positive definite matrices $0 < P^T = P \in \Re^{2n \times 2n}$, $0 \le Q^T = Q \in \Re^{n \times n}$, $0 < R^T = R \in \Re^{n \times n}$, an invertible matrix $N \in \Re^{m \times m}$, an arbitrary matrix $M = [M_1 \ \ M_2 \ \ M_3 \ \ M_4 \ \ M_5] \in \Re^{n \times (4n+m)}$ with $M_i \in \Re^{n \times n}$, $i = 1, \dots, 4$ and $M_5 \in \Re^{m \times n}$ and an appropriate matrix $F \in \Re^{m \times p}$ satisfying

$$\Omega < 0 \tag{3}$$

where $\Omega = \Psi + \Gamma_7^T FC\Gamma_1 + \Gamma_1^T C^T F^T \Gamma_7$ and $\Psi = \Gamma_1^T P \Gamma_3$

$$\begin{aligned} &+ \Gamma_3^T P \Gamma_1 + d\Gamma_2^T P(\Gamma_1 - \Gamma_4) + d(\Gamma_1 - \Gamma_4)^T P \Gamma_2 \\ &+ d^2 \Gamma_3^T Q \Gamma_3 - (\Gamma_1 - \Gamma_4)^T Q (\Gamma_1 - \Gamma_4) - 3(\Gamma_1 + \Gamma_4 - 2\Gamma_2)^T \\ &\times Q(\Gamma_1 + \Gamma_4 - 2\Gamma_2) + \Gamma_1^T R \Gamma_1 - \Gamma_4^T R \Gamma_4 + (M^T \Gamma_6 + \Gamma_7^T N \Gamma_8)\Gamma_9 \\ &+ \Gamma_9^T (M^T \Gamma_6 + \Gamma_7^T N \Gamma_8)^T \end{aligned}$$

with $\Gamma_1 = [\mathrm{I}_n \ \ 0_{n \times (3n+m)}]$,

$\Gamma_2 = [0_n \ \ \mathrm{I}_n \ \ 0_{n \times (2n+m)}]$, $\Gamma_3 = [0_{n \times 2n} \ \ \mathrm{I}_n \ \ 0_{n \times (n+m)}]$, $\Gamma_4 = [0_{n \times 3n} \ \ \mathrm{I}_n \ \ 0_{n \times m}]$, $\Gamma_5 = [0_{m \times 4n} \ \ \mathrm{I}_m]$, $\Gamma_6 = [\mathrm{I}_n \ \ 0_{n \times m}]$,

$\Gamma_7 = [\Lambda^T \ \ \Lambda^T \ \ \Lambda^T \ \ \Lambda^T \ \ \mathrm{I}_m]$, $\Lambda = [\mathrm{I}_m \ \ 0_{m \times (n-m)}]^T$,

$\Gamma_8 = [0_{m \times n} \ \ \mathrm{I}_m]$, $\Gamma_9 = [-\Gamma_3^T + \Gamma_1^T A^T + \Gamma_4^T A_d^T + \Gamma_5^T B^T \ \ -\Gamma_5^T]^T$

and I_j, $0_{k \times l}$ denote an identity matrix of *j* by *j* and a zero matrix of *k* by *l*, respectively, then the linear time delay