Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-10 (2016) 141–146

Lyapunov-Based Control of an Uncertain Euler-Lagrange System with Uncertain Time-Varying Input Delays without Delay Rate Constraints

S. Obuz * **A. Parikh** * **I. Chakraborty** * **W. E. Dixon** *

* *Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, USA (e-mail: serhat.obuz@ufl.edu, anuppari@ufl.edu, ichakraborty@ufl.edu, wdixon@ufl.edu).*

Abstract: A tracking controller is developed for a class of uncertain Euler-Lagrange systems with bounded external disturbances and an uncertain time-varying input delay. A novel filtered tracking error signal is designed to obtain a delay-free input signal in the closed-loop error system to facilitate the control design and analysis. The designed novel filtered tracking error consists of a finite integral over a constant estimated delay interval of the past control inputs as a means to compensate for the input delay. The maximum tolerable error between unknown time-varying delay and a constant estimate of the delay can be determined based on the selection of the control gains to provide uniformly ultimately bounded convergence of the tracking error to the origin. Lyapunov-Krasovskii functionals are used in the Lyapunov-based stability analysis.

1. INTRODUCTION

Control of input time delayed systems remains a challenging problem due to difficulties in predicting future states and compensating for disturbance effects in the input delayed dynamics. Much research in recent years has focused on developing controllers that provide stability for systems with delays in the closed-loop dynamics Richard [2003], Gu and Niculescu [2003], Niculescu and Gu [2004], Sipahi et al. [2011], Normey-Rico and Camacho [2008], Mazenc et al. [2008], Pepe and Jiang [2006]. Smith predictors Smith [1959], Artstein model reduction Artstein [1982], and the finite spectrum assignment Manitius and Olbrot [1979] have played an important role in the development of controllers that can handle the effects of input delays.

Linear systems with input delay have been extensively studied over the past ten years Choi and Lim [2010], Herrera et al. [2008], Polyakov et al. [2013], Chen and Zheng [2006], Li et al. [2014], Zhang and Li [2006], Yue [2004], Wang et al. [2013], Bresch-Pietri et al. [2012], Wang et al. [2005], Yue and Han [2005], Bresch-Pietri and Krstic [2009], Normey-Rico et al. [2009], Niculescu et al. [1996]. The work in Normey-Rico et al. [2009], Niculescu et al. [1996] developed robust controllers for compensating for input time delay effects in the uncertain linear plant dynamics, assuming the input delay is known. Motivated by the fact that delays can be difficult to measure, results such as Choi and Lim [2010], Herrera et al. [2008], Polyakov et al. [2013], Chen and Zheng [2006], Li et al. [2014], Zhang and Li [2006], Yue [2004], Wang et al. [2013], Bresch-Pietri et al. [2012], Wang et al. [2005], Yue and Han [2005], Bresch-Pietri and Krstic [2009] develop controllers that do not require exact knowledge of the delay.

Nonlinear systems with input delay are more challenging. Numerous studies have considered nonlinear systems with input delay Henson and Seborg [1994], Mazenc and Bliman [2006], Jankovic [2006], Sharma et al. [2011], Fischer et al. [2012], Obuz et al. [2012], Huang and Lewis [2003], Fischer et al. [2013, 2011], Nelson and Balas [2012], Balas and Nelson [2011], Bekiaris-Liberis and Krstic [2013], Mazenc and Niculescu [2011], Bresch-Pietri and Krstic [2014], Mazenc et al. [2013], Lozano et al. [2004]. In Sharma et al. [2011], Dinh et al. [2013], Fischer et al. [2012], Obuz et al. [2012], Huang and Lewis [2003], Fischer et al. [2013, 2011], Fischer [2012] controllers are developed for uncertain nonlinear systems while the delay measurement is assumed to be available. In practice, the duration of an input time delay can be challenging to determine for some applications. The authors in Balas and Nelson [2011], Bekiaris-Liberis and Krstic [2013], Mazenc and Niculescu [2011], Bresch-Pietri and Krstic [2014] develop controllers for plants with nonlinear dynamics and an unknown input delay. For example, the controller in Bresch-Pietri and Krstic [2014] is designed to compensate for an arbitrarily large, uncertain, constant input delay with known bounds by using an adaptive prediction based technique for a class of nonlinear systems with exact model knowledge of the dynamics. This results in global asymptotic convergence for the case when the full-state is available for feedback and local regulation for the case of missing measurements of the actuator state by using an adaptive based estimate instead of the actuator state. However, the controllers in Balas and Nelson [2011], Bekiaris-Liberis and Krstic [2013], Mazenc and Niculescu [2011], Bresch-Pietri and Krstic [2014] require exact model knowledge of the nonlinear dynamics. Such results specifically exploit exact model knowledge as a means to predict the evolution of the states. Parameter uncertainty in nonlinear systems with additive disturbances significantly complicates the ability to predict the state

⋆

2405-8963 © 2016, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.07.506

transition, especially without knowing the time-varying input delay. Regardless of the difficulty in predicting the state transition over the time delay interval, there remains a need for a tracking controller that can compensate for the uncertain input delay despite uncertainty in the dynamic model and additive disturbances.

Recently, Fischer et al. designed a continuous robust controller that is robust to the time input delay of an uncertain Euler-Lagrange system with external disturbances Fischer et al. [2012], by assuming that the input delay is known, the first and second derivative of the delay is bounded by a known constant, and the inertia matrix of the Euler-Lagrange dynamics is known. The techniques used in this study build upon, but significantly extended, our previous work in Fischer et al. [2012]. The contribution of this work is that the first and second time derivative of the input delay do not have to be bounded (i.e., delay rate constraints are eliminated) due to the development of a constant estimated delay instead of the actual time-varying known/estimated delay in the novel error signal, and the measurement delay and inertia matrix do not have to be known. Novel Lyapunov-Krasovskii functionals that are not functions of time-varying delay are used in the stability analysis to eliminate requirements of the slowly-varying delay rate. The result is achieved by using a novel filtered error signal that facilitates compensating for the uncertain, time varying input delay in the uncertain Euler-Lagrange system with additive disturbances. In Fischer et al. [2012], a filtered error signal developed based on the finite integral of the input signals over the known delay interval is used to obtain a delay-free control signal in the closed-loop error system, but the computation of the filtered error signal requires that the delay to be known. To cope with the lack of delay knowledge as well as an upper bound on the first and second time derivative of the delay, a novel error signal is designed using the past states in a finite integral over a constant estimated delay interval. Based on the development of this modified filtering error signal, a novel Lyapunov-based stability analysis is developed by using Lyapunov-Krasovskii functionals to prove uniformly ultimate boundedness of the error signals. As opposed to previous results in Fischer et al. [2012], in this paper, an estimate of the input delay is used in the control, and only bounds on the delay are required to be known. Additionally, the maximum tolerable delay error between the estimate of the input delay and actual input delay can be determined based on selection of control gains in the controller.

2. DYNAMIC MODEL AND PROPERTIES

Consider a class of Euler-Lagrange systems defined by

$$M(q)\,\ddot{q}+V_m(q,\dot{q})\,\dot{q}+G(q)+F(\dot{q})+d\,(t)=u\,(t-\tau\,(t))\,, \quad (1)$$

where $q,\dot{q},\ddot{q}\in\mathbb{R}^n$ denote the generalized position, velocity, and acceleration, $M:\mathbb{R}^n\to\mathbb{R}^{n\times n}$ is an uncertain generalized inertia matrix, $V_m:\mathbb{R}^{2n}\to\mathbb{R}^{n\times n}$ is an uncertain generalized centripetal-Coriolis matrix, $G:\mathbb{R}^n\to\mathbb{R}^n$ denotes an uncertain generalized gravity vector, $F:\mathbb{R}^n\to\mathbb{R}^n$ denotes uncertain generalized friction, $d:[t_0,\infty)\to\mathbb{R}^n$ is an uncertain exogenous disturbance (e.g., unmodeled effects which could explicitly depend on time), $u\,(t-\tau\,(t))\in\mathbb{R}^n$ represents the generalized delayed input control vector, $\tau:[t_0,\infty)\to\mathbb{R}$ is an uncertain non-negative time-varying delay, and t_0 is the initial time.

The subsequent development is based on the assumption that $q,\dot{q}$ are measurable. Throughout the paper, delayed functions are denoted as

$$h_\tau\,(t)\triangleq\begin{cases}h\,(t-\tau) & t-\tau\geq t_0\\ 0 & t-\tau<t_0.\end{cases}$$

In addition, the dynamics of the system in (1) satisfies the following assumptions and properties.

The functions M, V_m, G and F have bounded first order partial derivatives provided that the functions q, $\dot{q}$, $\ddot{q}$ are bounded.

Assumption 1. The functions M, V_m, G and F have bounded first order partial derivatives [1] provided that the functions q, $\dot{q}$, $\ddot{q}$ are bounded.Makkar et al. [2007].

Assumption 2. The nonlinear exogenous disturbance term and its first time derivative (i.e., d, $\dot{d}$) exist and are bounded by known positive constants Patre et al. [2008, 2011], Sharma et al. [2012].

Assumption 3. The reference trajectory $q_d\in\mathbb{R}^n$ is designed such that $q_d,\dot{q}_d,\ddot{q}_d$ exist and are bounded by known positive constants.

Assumption 4. The input delay is differentiable and bounded as $\tau\,(t)<\Gamma$, $\forall t\in\mathbb{R}$, where $\Gamma\in\mathbb{R}$ is a known positive constant. Moreover, a positive constant estimate $\hat{\tau}\in\mathbb{R}$ is sufficiently accurate in sense that $|\tilde{\tau}|\leq\bar{\tilde{\tau}}$ where $\tilde{\tau}\triangleq\tau-\hat{\tau}$ and $\bar{\tilde{\tau}}\in\mathbb{R}$ is a known constant [2]. Additionally, the system in (1) does not escape to infinity during the time interval $[t_0,t_0+\Gamma]$is assumed.

Assumption 5. The inertia matrix M is symmetric positive-definite, and satisfies the following inequality:

$$\underline{m}\,\|\xi\|^2\leq\xi^T M\xi\leq\overline{m}\,\|\xi\|^2\,,\quad\forall\xi\in\mathbb{R}^n,$$

where $\underline{m},\overline{m}\in\mathbb{R}$ are known positive constants and $\|\cdot\|$ denotes the standard Euclidean norm.

3. CONTROL OBJECTIVE

The objective is to develop a continuous controller which ensures that the generalized state q of the input-delayed system in (1) tracks a reference trajectory, q_d, despite uncertainties and additive disturbances in the dynamics. To quantify the control objective, a tracking error, denoted by $e\in\mathbb{R}^n$, is defined as

$$e\triangleq q_d-q. \quad (2)$$

To facilitate the subsequent analysis, a measurable auxiliary tracking error, denoted by $r\in\mathbb{R}^n$, is defined as

[1] The bounds of $M\,(q\,(t))$, $\frac{\partial M(q(t))}{\partial q}$, $\frac{\partial M(q(t))}{\partial t}$, $V_m\,(q\,(t)\,,\dot{q}\,(t))$, $\frac{\partial V_m(q(t),\dot{q}(t))}{\partial q}$, $\frac{\partial V_m(q(t),\dot{q}(t))}{\partial \dot{q}}$, $\frac{\partial V_m(q(t),\dot{q}(t))}{\partial t}$, $G\,(q\,(t))$, $\frac{\partial G(q(t))}{\partial q}$, $\frac{\partial G(q(t))}{\partial t}$ and $F\,(\dot{q}\,(t))$, $\frac{\partial F(\dot{q}(t))}{\partial \dot{q}}$, $\frac{\partial F(\dot{q}(t))}{\partial t}$ are assumed to be known. To satisfy Assumption 1, the aforementioned functions do not need to be bounded for all time. Assumption 1 only requires that provided the functions q, $\dot{q}$, $\ddot{q}$ are bounded, then the functions are uniformly bounded in t.

[2] Since the maximum tolerable error, $\bar{\tilde{\tau}}$, can be determined based on the selection of control gains, and the estimate of the actual delay, $\hat{\tau}$, is known, the maximum tolerable input delay can be determined.