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Input Delay Compensation in Multi-Agent Systems via Piecewise Constant Control

Anton Ponomarev * **Hai-Tao Zhang** **

* *Department of Control Theory, Saint Petersburg State University, Saint Petersburg 198504, Russia (e-mail: a.ponomarev@spbu.ru)*
** *School of Automation, Huazhong University of Science and Technology, Wuhan 430074, P.R. China (e-mail: zht@mail.hust.edu.cn)*

Abstract: We consider a multi-agent system with time-invariant topology containing several agents governed by similar linear time-invariant ODEs with constant delay in actuation. Piecewise constant (PWC) control with sampling time larger than the delay value is applied. Assuming the delay-free version of the system can be brought to consensus by the PWC feedback, we design a predictive PWC feedback achieving consensus in the system with delayed input. During prediction, PWC structure is employed to determine future input to the system from its past output. This input is then used to predict the future state and compensate for the delay.

Keywords: Linear control systems, multi-agent systems, sampling control, delay compensation, predictive control.

1. INTRODUCTION

Multi-agent system (MAS) is a group of dynamical objects (agents) where each agent can only communicate with a certain subset of other agents. See Martínez et al. (2007) for an overview of this topic. Distributed structure of MAS poses the problem of designing a control algorithm which, when employed *locally* by each agent, leads to synchronized *group* behavior (Olfati-Saber (2006); Jadbabaie et al. (2003); Zhang et al. (2008); Zhang and Chen (2014)). The kinds of synchronized behavior usually considered include leader following (Zhu and Cheng (2010)), pattern formation (Dong et al. (2014, 2016)), rendezvous (Cortés et al. (2006)), etc. The one being studied in this paper is called *consensus* and refers to all agents converging to each other disregarding whether they settle at a stationary point or keep moving.

Delays may occur in the feedback loop if a substantial amount of time is required to gather signals from the sensors, compute the desired control and communicate it to the actuators. In the present paper we consider only input delay – when the system is actuated by an "outdated" value of the input.

When ordinary PID controllers are used in systems with input delays, the closed loop becomes a time-delay system. Stability of time-delay systems is becoming a fairly well-studied area, particularly due to the use of Lyapunov theory (Kharitonov (2013)), but some control-related areas, e.g., optimal feedback synthesis, are yet to evolve.

There is a control methodology which aims at eliminating input delays from the system, thus enabling a range of delay-free control methods to be used. It is called *delay compensation* or *predictive feedback* (PF) which means that the controller predicts what the state of the system is going to be at the future moment when the currently chosen control is to be actually applied. The method was introduced in Manitius and Olbrot (1979), Kwon and Pearson (1980), and Artstein (1982) for linear ODE systems. Today it is developed, e.g., for systems with uncertain or time-varying delay values (Bresch-Pietri and Krstic (2010); Bresch-Pietri et al. (2012)), state-dependent delays (Bekiaris-Liberis and Krstic (2013)), nonlinear systems (Krstic (2010); Ponomarev (2015)), and systems with both state and input delays (Kharitonov (2014)). In the field of MAS, however, PF is a relatively new topic.

Due to the distributed nature of MAS, one agent only knows some of the other agents' states. It cannot easily predict future states of other agents because it does not know the controls the other ones are going to apply in the future. This problem hinders straightforward use of PF in MAS.

Recently, *truncated predictor feedback* (TPF) was developed for MAS in Zhou and Lin (2014). The method makes use of the *small gain feedback* theory (Lin (1999)) in the following way. State prediction for linear systems (expressed by the *variation of constants* formula) contains two terms: one dependent on the current system state, and another one dependent on the past controls. Having chosen a "small enough" feedback gain, one can drop the control-dependent part because it is of higher order of smallness than the state-dependent part. As a result, PF turns into a proportional regulator.

In this paper we suggest *piecewise constant* (PWC) control as a remedy to the problem of state prediction. PWC control (also known as sampled-data periodic feedback)

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is well-known from the literature (Karafyllis and Krstic (2012); Mondie et al. (2002)). The novelty of our paper is in applying it for the specific purpose: to enable delay compensation in multi-agent systems. For example, let the current time instant be t. Knowing that all agents' controls are kept constant during some interval $[t-a; t+b]$ $(a, b > 0)$, each agent can determine others' controls by analyzing the system's states on $[t-a; t]$ and then predict future states on $[t; t+b]$.

Let us compare our approach to TPF. Our advantages are:

- TPF admits only "small enough" feedback gains whereas our method suits any choice of control gains, enabling more aggressive or effective control;
- TPF requires that the open loop system be at most *polynomially* unstable whereas our method is applicable to *exponentially* unstable systems.

However, at the same time we recognize the PWC style of control as the main drawback of our method due to the following points:

- control sampling time must be larger than the delay value, therefore, our approach may be undesirable for very large delays as it would lead to overly discretized control and deteriorate the closed-loop performance or even make the system uncontrollable;
- one should be aware that when random disturbances are present in the system, fair prediction/estimation of the future output requires larger sampling time (specifically, the interval of integration in the feedback (14) must be long enough) and is a possible downside.

Though in this paper we focus on constant delay, our method can be used similarly for bounded time-varying delays. For further comments on the future development of our technique, see "Conclusion".

2. PROBLEM STATEMENT

Consider the *multi-agent system* (MAS) of N similar agents governed by equations

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t-h), \quad i = 1, 2, \dots, N, \quad t \geq 0 \quad (1)$$

where $x_i \in R^n$, $u_i \in R^m$. Here h is a constant input delay. Initial input for $(t < 0)$ can be arbitrary.

Distributed multi-agent nature of the MAS is enforced by the rule that the i-th agent's controller u_i cannot sense other agents' states directly but can only operate on the value

$$y_i(t) = \sum_{j=1}^{N} \alpha_{ij}\big(x_i(t) - x_j(t)\big). \quad (2)$$

It is a weighted sum of other agents' states respective to the i-th agent. If $\alpha_{ij} = 0$ then the i-th agent cannot sense the j-th agent at all.

The problem is to design feedbacks $u_i(y_i)$ which achieve consensus in the MAS. We define consensus as the closed-loop behavior when all agents' states converge to each other:

$$\lim_{t\to\infty} \big(x_i(t) - x_N(t)\big) = 0, \quad i = 1, 2, \dots, N-1. \quad (3)$$

3. PRELIMINARIES

3.1 Piecewise constant control

PWC type of control is an important part of our approach. Let us choose $T_s > h$ and consider the control

$$u_i(t) \equiv u_i(kT_s) \quad \forall t \in [kT_s; (k+1)T_s), \quad k = 0, 1, \dots \quad (4)$$

Due to the delay, input $u_i(kT_s)$ will actuate the system on the interval $[kT_s + h, (k+1)T_s + h)$, so the T_s-sampled dynamics of x_i are described as

$$x_i\big((k+1)T_s + h\big) = \bar{A}x_i(kT_s + h) + \bar{B}u_i(kT_s) \quad (5)$$

where

$$\bar{A} = e^{AT_s}, \quad \bar{B} = \int_0^{T_s} e^{A(T_s-\theta)}\, d\theta B. \quad (6)$$

3.2 Delay-free consensus

Consider the delay-free case $(h = 0)$:

$$x_i\big((k+1)T_s\big) = \bar{A}x_i(kT_s) + \bar{B}u_i(kT_s). \quad (7)$$

Suppose that under the feedbacks

$$u_i(kT_s) = Fy_i(kT_s), \quad i = 1, 2, \dots, N \quad (8)$$

system (7) reaches consensus. For the reference, we write down the closed loop (7), (8):

$$x_i\big((k+1)T_s\big) = \bar{A}x_i(kT_s) + \bar{B}Fy_i(kT_s). \quad (9)$$

3.3 Delay compensation principle

Suppose $h \neq 0$. Usage of the feedback (8) would introduce time delay into the closed loop thus hindering stability analysis and/or appropriate choice of the feedback gain F. To compensate for the delay, it is desirable to use $y_i(kT_s + h)$ instead of $y_i(kT_s)$ in (8):

$$u_i(kT_s) = Fy_i(kT_s + h), \quad i = 1, 2, \dots, N. \quad (10)$$

The result is that the closed loop (5), (10) is delay-free:

$$x_i\big((k+1)T_s + h\big) = \bar{A}x_i(kT_s + h) + \bar{B}Fy_i(kT_s + h), \quad i = 1, 2, \dots, N. \quad (11)$$

4. MAIN RESULT

The problem with using delay compensation feedback (10) is:

$$\text{how to find } y_i(kT_s + h) \text{ at time } kT_s? \quad (12)$$

The common approach is *predictive feedback* (PF) which employs the system's model (1) to predict future outputs y_i, but for MAS it is not easy to do because the i-th agent's observations y_i depend on other agents' inputs which are unavailable to i. However, thanks to usage of PWC control (4) we can write

$$\dot{y}_i(t) = Ay_i(t) + w_i\big((k-1)T_s\big) \quad \forall t \in [(k-1)T_s + h; kT_s + h) \quad (13)$$

where $w_i((k-1)T_s)$ is a vector unknown to the i-th agent. In the problem (12) we know $y_i(t)$ for $t \in [(k-1)T_s + h; kT_s]$. On that interval, the same constant $w_i((k-1)T_s)$ was used as will be used on $[kT_s; kT_s + h]$. Thus, we can recover the value of w_i from the record of y_i on $[(k-1)T_s + h; kT_s]$ and use it to predict y_i over $[kT_s; kT_s + h]$. This idea is the foundation of the following result.