Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-10 (2016) 171–176

Dynamics of Heterogeneous Connected Vehicle Systems

Nan I. Li and Gábor Orosz

Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI, 48109, USA (e-mail: nanli@umich.edu, orosz@umich.edu)

Abstract: In this paper, we propose a method to analyze the longitudinal dynamics of heterogeneous connected vehicle systems including human-driven vehicles and vehicles driven by connected cruise control. Human reaction time and digital sampling time are incorporated in the models. Conditions of plant stability and head-to-tail string stability are presented.

Keywords: vehicle-to-vehicle (V2V) communication, heterogeneity, time delay, string stability

1. INTRODUCTION

Advanced driver assistance systems (ADAS) have been used to improve vehicle safety and passenger comfort in the last couple of decades. Vehicle-to-vehicle (V2V) communication has the potential to further enhance the performance of these systems by allowing the vehicle to monitor a larger traffic environment; see (Kianfar et al., 2012), (di Bernardo et al., 2015), and (Alam et al., 2015). Recently, connected cruise control (CCC) was proposed to regulate the longitudinal motion of vehicles, which exploits V2V information broadcast by multiple vehicles ahead and allows high flexibility in the structure of vehicle networking; see (Orosz, 2014). While V2V-based ADAS systems are traditionally applied for platoons composed of digitally controlled vehicles, CCC can be used to improve traffic conditions even with low penetration of CCC vehicles within the human-driven traffic flow.

When mixing CCC vehicles to the flow of human-driven vehicles, a hybrid system is created since human-driven vehicles operate in continuous time, while CCC vehicles are controlled by digital controllers in discrete time. In previous works, such systems were analyzed either by converting everything into discrete time by discretizing the continuous-time dynamics (Qin et al., 2015); or by converting everything into continuous time by approximating the time-varying delay imposed by the discrete-time dynamics with constant time delay (Ge and Orosz, 2014). Both of these are approximations of the true dynamics that will be considered in this paper. In particular, we are interested in the following performance measures: plant stability and string stability. Plant stability indicates the ability of a vehicle to approach steady state when no disturbances are imposed by other vehicles. On the other hand, string stability indicates the ability of a vehicle to attenuate disturbances imposed by the vehicles ahead. String stability is typically a stronger condition and in this paper we mainly focus on string stability.

The paper is organized as follows. In Section 2, a generalized modeling framework for the longitudinal dynamics of connected vehicle systems containing human-driven vehicles and CCC vehicles is presented, together with criteria for plant stability and head-to-tail string stability. In Section 3, we derive the formulae to analyze string stability of human-driven vehicle networks, CCC vehicle networks, and general heterogeneous connected vehicle systems that contain both human-driven and CCC vehicles. In Section 4, we investigate two connected vehicle systems as case studies to validate our formulae and make some comparisons to the two approximation methods. Finally, we conclude our paper in Section 5.

2. DYNAMICS AND STABILITY

In this section, a general modeling framework for the longitudinal dynamics of human-driven vehicles and CCC vehicles is presented.

2.1 Longitudinal Dynamics of Human-driven Vehicles

We assume that a human driver can monitor the motion of the vehicle immediately ahead and respond to stimuli like the headway h, the velocity v and the velocity of the car ahead v_1 with a reaction time delay; see Fig. 1(a). We model human drivers using the continuous-time deterministic system

$$\begin{aligned} \dot{h}(t) &= v_1(t) - v(t), \\ \dot{v}(t) &= \alpha_{\rm h}\big(V(h(t-\tau)) - v(t-\tau)\big) \\ &\quad + \beta_{\rm h}\big(v_1(t-\tau) - v(t-\tau)\big), \end{aligned} \tag{1}$$

where the dot stands for differentiation with respect to time t, $\alpha_{\rm h}$ represents the gain to match the actual velocity to a distance dependent reference velocity, while $\beta_{\rm h}$ represents the gain to match the velocity to that of the vehicle ahead. Also, τ represents human reaction time, which is typically in the range 0.4∼1.0 [s]. The function $V(h)$ denotes the range policy, which gives the reference velocity as a function of the headway h. In particular, we assume the monotonically increasing range policy function

$$V(h) = \begin{cases} 0 & \text{if} \quad h \le h_{\rm st}, \\ v_{\max}\big[1 - \cos\big(\pi \frac{h - h_{\rm st}}{h_{\rm go} - h_{\rm st}}\big)\big] & \text{if} \quad h_{\rm st} < h < h_{\rm go}, \\ v_{\max} & \text{if} \quad h \ge h_{\rm go}, \end{cases} \tag{2}$$

10.1016/j.ifacol.2016.07.520

which represents the driver's intention to keep a larger distance with increasing speed; see (Ge and Orosz, 2014).

In this paper we investigate dynamics in the vicinity of the equilibrium

$$h(t) \equiv h^*, \quad v_1(t) \equiv v(t) \equiv v^* = V(h^*). \tag{3}$$

One may define the perturbations

$$\tilde{h}(t) = h(t) - h^*, \quad \tilde{v}_1(t) = v_1(t) - v^*, \quad \tilde{v}(t) = v(t) - v^*, \tag{4}$$

and linearize (1) about (3) to obtain

$$\dot{x}(t) = A_{\rm h0}x(t) + A_{\rm h1}x(t-\tau) + B_{\rm h0}\tilde{v}_1(t) + B_{\rm h1}\tilde{v}_1(t-\tau), \tag{5}$$

where $x = [\tilde{h} \ \tilde{v}]^{\rm T}$ and the matrices are given by

$$A_{\rm h0} = \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \qquad A_{\rm h1} = \begin{bmatrix} 0 & 0 \\ \alpha_{\rm h}N & -(\alpha_{\rm h} + \beta_{\rm h}) \end{bmatrix},$$
$$B_{\rm h0} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad B_{\rm h1} = \begin{bmatrix} 0 \\ \beta_{\rm h} \end{bmatrix}, \tag{6}$$

and $N = V'(h^*)$ is the derivative of range policy (2) at the equilibrium. Note that (5) is a linear system with constant delay that appears both in the input and in the state.

We are interested in the longitudinal velocity of the vehicle, so we define the output

$$\tilde{v} = Cx, \qquad C = [0 \ 1]\,. \tag{7}$$

For a linear time-invariant (LTI) system, we can use a transfer function to represent the dynamic relationship between the input and the output. Taking the Laplace transform of (5,7) with zero initial condition, we obtain

$$\tilde{V}(s) = T^{\rm h}(s)\tilde{V}_1(s) \tag{8}$$

where $\tilde{V}(s)$ and $\tilde{V}_1(s)$ represent the Laplace transform of $\tilde{v}(t)$ and $\tilde{v}_1(t)$, respectively, and the transfer function is

$$T^{\rm h}(s) = C\big(sI - A_{\rm h0} - A_{\rm h1}{\rm e}^{-\tau s}\big)^{-1}\big(B_{\rm h0} + B_{\rm h1}{\rm e}^{-\tau s}\big). \tag{9}$$

When driving the system with periodic input $\tilde{v}_1(t) = v_1^{\rm amp}\sin(\omega t)$, the steady state output becomes $\tilde{v}^{\rm ss}(t) = |T^{\rm h}({\rm j}\omega)|v_1^{\rm amp}\sin(\omega t + \measuredangle T^{\rm h}({\rm j}\omega))$, where $|\cdot|$ and $\measuredangle$ denote the magnitude and the angle of a complex number.

2.2 Longitudinal Dynamics of CCC Vehicles

We assume that a CCC vehicle can monitor the positions and velocities of multiple vehicles ahead through V2V communication and use this information to control its own motion. Fig. 1(b) shows a scenario where the CCC vehicle monitors the motion of n vehicles ahead, which may be human-driven or CCC vehicles. In particular, we assume that it monitors the headway h and the velocities $v_1, ..., v_n$. We also assume that the clocks of the connected vehicles are synchronized and no packets are dropped.

We assume that the CCC vehicle uses a similar control algorithm as the human drivers but applies a zero-order hold (ZOH). Thus, at the time interval $t \in [k\Delta t, (k+1)\Delta t)$ its dynamics is governed by

$$\begin{aligned} \dot{h}(t) &= v_1(t) - v(t), \\ \dot{v}(t) &= u\big((k-1)\Delta t\big), \\ u(t) &= \alpha\big(V(h(t)) - v(t)\big) + \sum_{i=1}^{n}\beta_i\big(v_i(t) - v(t)\big), \end{aligned} \tag{10}$$

where α is the gain for the difference between the velocity and the reference velocity given by range policy (2), while

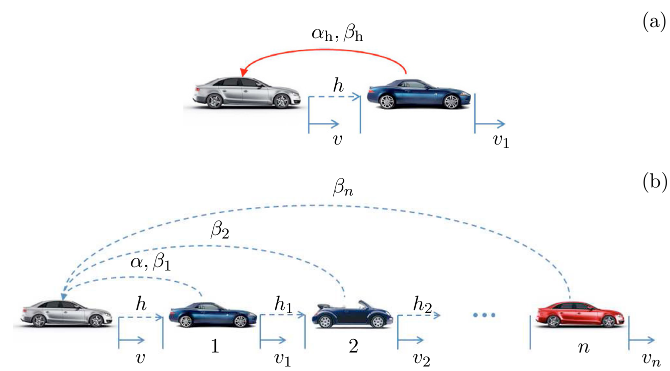

Fig. 1. (a) Human-driven vehicle monitors the vehicle immediately ahead. (b) CCC vehicle at the tail receives information from n vehicles ahead. The velocities and the headways are denoted by $v, v_1, ..., v_n$ and $h, h_1, ..., h_{n-1}$, respectively. The gain parameters are displayed along the communication links.

β_i, $i = 1, ..., n$ are the gains for the velocity differences. We remark that if vehicle i does not broadcast its velocity, we set the corresponding gain $\beta_i = 0$. Finally, Δt represents the sampling period of the digital controller, which is set to be larger than the time needed for sampling, broadcasting, receiving and processing the information. The sampling frequency should satisfy the Nyquist criterion, i.e., $\frac{2\pi}{\Delta t} > 2\omega_{\max}$, where $\omega_{\max}$ is the largest meaningful angular frequency for longitudinal vehicle dynamics. Setting $\Delta t = 0.1$ [s], which is common in V2V communication, the Nyquist criterion is typically satisfied.

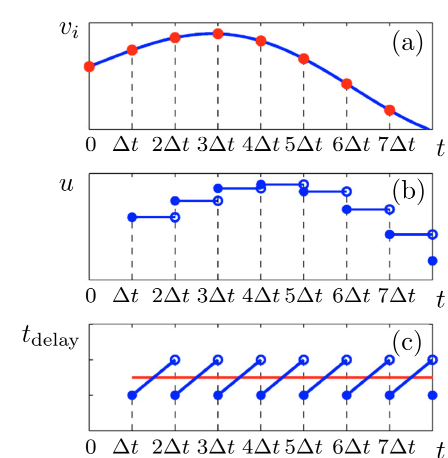

Fig. 2. (a) The velocity of vehicle i (solid blue line) as a function of time with sampled data (red dots). (b) The control signal of a CCC vehicle as a function of time. (c) The change of the delay in the control loop as a function of time, where the average delay is represented by the red horizontal line.

Again, we consider the dynamics about equilibrium

$$h(t) \equiv h^*, \quad v(t) \equiv v_i(t) \equiv v^* = V(h^*), \quad i = 1, ..., n, \tag{11}$$

define the perturbations

$$\begin{aligned} \tilde{h}(t) &= h(t) - h^*, \quad \tilde{v}(t) = v(t) - v^*, \\ \tilde{v}_i(t) &= v_i(t) - v^*, \quad i = 1, ..., n, \end{aligned} \tag{12}$$

and integrate (10) between $k\Delta t$ and $(k+1)\Delta t$ to obtain the linear difference equation