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## Controller Design for Neutral Time-Delay Controller Design for Neutral Time-Delay ontroner Design for Neutral Time-Dela<br>Systems by Nonsmooth Optimization  $^{\star}$ Controller Design for Neutral Time-Delay Controller Design for Neutral Time-Delay

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A controller design algorithm based on a nonsmooth optimization approach is proposed. An A controller design algorithm based on a honshooth optimization approach is proposed. All<br>initialization procedure for the non-convex optimization problem is also presented. Finally, the initialization procedure for the non-convex optimization problem is also presented. Finally, the proposed approach is demonstrated by an example. proposed approach is demonstrated by an example. *∗ smozer@anadolu.edu.tr*<br>
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Abstract: Strong stabilization of linear time-invariant neutral time-delay systems is considered. initialization procedure for the non-convex optimization problem is also presented. Finally, the non-control by an example

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Keywords: Time-delay systems; neutral systems; strong stabilization; nonsmooth optimization. Keywords: Time-delay systems; neutral systems; strong stabilization; nonsmooth optimization. Keywords: Time-delay systems; neutral systems; strong stabilization; nonsmooth optimization.

## 1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION

Eigenvalue-based methods have recently become popular Eigenvalue-based methods have recently become popular in the stabilization of both neutral and retarded linear time-invariant (LTI) time-delay systems (e.g., see Michiels time-invariant (LTI) time-delay systems (e.g., see Michiels and Niculescu (2007) and references therein). Stabilization of a neutral time-delay system, however, is much harder of a neutral time-delay system, however, is much harder than that of a retarded time-delay system. There are two than that of a retarded time-delay system. There are two main reasons for that. First of all, although a retarded LTI time-delay system has only a finite number of modes in any time-delay system has only a finite number of modes in any given right-half complex plane, this is not true for a neutral given right-half complex plane, this is not true for a neutral system (Niculescu (2001)). Secondly, although the modes system (Niculescu (2001)). Secondly, although the modes of a retarded LTI time-delay system varies continuously of a retarded LTI time-delay system varies continuously, frequency modes of a neutral LTI time-delay system are frequency modes of a neutral LTI time-delay system are sensitive to infinitesimal changes in the time-delays. To sensitive to infinitesimal changes in the time-delays. To overcome the latter difficulty, the concept of strong sta-overcome the latter difficulty, the concept of strong stability was introduced by Hale and Verduyn-Lunel (1993). A time-delay system is said to be strongly stable if it is A time-delay system is said to be strongly stable if it is stable and remains stable under infinitesimal changes in the time-delays. One of the eigenvalue-based stabilization the time-delays. One of the eigenvalue-based stabilization methods proposed for LTI time-delay systems is the so-methods proposed for LTI time-delay systems is the socalled *continuous pole placement* method, which was originally proposed by Michiels et al. (2002) for retarded timedelay systems. This approach was then extended to neutral delay systems. This approach was then extended to neutral time-delay systems by Michiels and Vyhlidal (2005), where time-delay systems by Michiels and Vyhlidal (2005), where strong stability was considered. This method has also been strong stability was considered. This method has also been extended for decentralized controller design by Erol and ˙ Iftar (2014, 2015). As an alternative to the continuous ˙ Iftar (2014, 2015). As an alternative to the continuous pole placement method, a nonsmooth optimization-based pole placement method, a nonsmooth optimization-based method was proposed by Vanbiervliet et al. (2008) for method was proposed by Vanbiervliet et al. (2008) for retarded time-delay systems. This method was then employed by Vyhlidal et al. (2011) to design strongly stabilizing state-derivative controllers for retarded time-delay lizing state-derivative controllers for retarded time-delay systems. Although, the given systems were assumed to be systems. Although, the given systems were assumed to be retarded in Vyhlidal et al. (2011), the closed-loop system retarded in Vyhlidal et al. (2011), the closed-loop system becomes neutral due to the structure of the controller becomes neutral due to the structure of the controller used. Finally, strong stabilization of neutral time-delay systems, written in the form of delay-differential-algebraic Eigenvalue-based methods have recently become popular Eigenvalue-based methods have recently become popular in the stabilization of both neutral and retarded linear in the stabilization of both neutral and retarded linear Eigenvalue-based methods have recently become popular time-invariant (LTI) time-delay systems (e.g., see Michiels time-invariant (LTI) time-delay systems (e.g., see Michiels in the stabilization of both neutral and retarded linear and Niculescu (2007) and references therein). Stabilization and Niculescu (2007) and references therein). Stabilization time-invariant (LTI) time-delay systems (e.g., see Michiels and Niculescu (2007) and references therein). Stabilization<br>of a neutral time-delay system, however, is much harder of a neutral time-delay system, however, is much harder<br>than that of a retarded time-delay system. 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To<br>overcome the latter difficulty, the concept of *strong sta*bility was introduced by Hale and Verduyn-Lunel (1993).  $\mu$  time-delay system is said to be strongly stable if it is State and remains stable under infinite intervention.<br>A time-delay system is said to be strongly stable if it is stable and remains stable under immitesimal changes in<br>the time-delays. One of the eigenvalue-based stabilization methods proposed for LTI time-delay systems is the so-methods proposed for LTI time-delay systems is the so-the time-delays. One of the eigenvalue-based stabilization methods proposed for LTI time-delay systems is the so-<br>called *continuous pole placement* method, which was origicalled *continuous pole placement* inferiod, which was originally proposed by Michiels et al. (2002) for retarded time-<br>delay systems. This approach was then extended to neutral nally proposed by Michiels et al. (2002) for retarded time-<br>delay systems. 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Finally, strong stabilization of neutral time-delay<br>systems, written in the form of delay-differential Available coline at www.science<br> **ScienceDirec C**<br> **June 22-24, 2016**<br> **ScienceDirec C**<br> **SCienceDirec C**<br> **INSEE INSEE IFAC-Experimental at the COMPREMIC CONTRESS CONTRESS CONTRESS (<br>
<b>SCIENCE CONTRESS)**<br> **SCIENCE CONTRE** time-delay system has only a finite number of modes in any<br>given right-half complex plane this is not true for a neutral  $\overline{a}$ 

equations, was considered by Michiels (2011). It should, equations, was considered by Michiels (2011). It should, however, be noted that since the optimization problem is not convex, the choice of the initial parameters plays is not convex, the choice of the initial parameters plays an important role in the success of those algorithms. In an important role in the success of those algorithms. In the present work, we employ the nonsmooth optimization approach to strongly stabilize a given LTI neutral time-approach to strongly stabilize a given LTI neutral timedelay system along the lines of Michiels (2011). As a novel delay system along the lines of Michiels (2011). As a novel contribution, we propose an initialization procedure for the optimization problem. Furthermore, as in  $\ddot{\mathrm{O}}$ zer and Iftar (2015), where decentralized controller design by nonsmooth optimization for retarded time-delay systems was smooth optimization for retarded time-delay systems was considered, we structure the controller matrices in certain canonical forms. The problem is stated in Section 2 and canonical forms. The problem is stated in Section 2 and its proposed solution is given in Section 3. An example is its proposed solution is given in Section 3. An example is presented in Section 4 and some concluding remarks are<br>presented in Section 4 and some concluding remarks are given in Section 5. given in Section 5. given in Section 5. given in Section 5. equations, was considered by Michiels (2011). It should, equations, was considered by Michiels (2011). It should, however, be noted that since the optimization problem however, be noted that since the optimization problem equations, was considered by Michiels (2011). It should, equations, was considered by Michiels (2011). It should, however, be noted that since the optimization problem is not convex, the choice of the initial parameters plays an important role in the success of those algorithms. In an important role in the success of those algorithms. In is not convex, the choice of the initial parameters plays the present work, we employ the nonsmooth optimization the present work, we employ the nonsmooth optimization an important role in the success of those algorithms. In approach to strongly stabilize a given LTI neutral time-approach to strongly stabilize a given LTI neutral time-the present work, we employ the nonsmooth optimization delay system along the lines of Michiels (2011). As a novel delay system along the lines of Michiels (2011). As a novel approach to strongly stabilize a given LTI neutral timedelay system along the lines of Michiels (2011). As a novel<br>contribution, we propose an initialization procedure for the optimization procedure for<br>the optimization problem. Furthermore, as in Ozer and the optimization, we propose an initialization procedure for<br>the optimization problem. Furthermore, as in Özer and<br>Iftar (2015), where decentralized controller design by nonthe optimization problem. Furthermore, as in Ozer and iftar (2015), where decentralized controller design by nonsmooth optimization for retarded time-delay systems was smooth optimization for retarded time-delay systems was Iftar (2015), where decentralized controller design by nonsmooth optimization for retarded time-delay systems was<br>considered, we structure the controller matrices in certain canonical forms. The problem is stated in Section 2 and canonical forms. The problem is stated in Section 2 and considered, we structure the controller matrices in certain  $\alpha$  canonical forms. The problem is stated in Section 2 and its proposed solution is given in Section 2 and<br>its proposed solution is given in Section 3. An example is<br> **ScienceDirect**<br>
Schemes and the set of the set of the set of the set of the set of the set of the set of the set of the set of the set of the set of the set of the set of the set of the set of the set of the set of the s  $\overline{a}$ presented in Section 4 and some concluding remarks are<br>given in Section 5

Throughout the paper,  $\mathbb{N}$ ,  $\mathbb{R}$ , and  $\mathbb{C}$  denote the sets of nonnegative integers, real numbers, and complex numbers, negative integers, real numbers, and complex numbers, respectively. For  $s \in \mathbb{C}$ , Re $(s)$  and Im $(s)$  denote the real and imaginary parts of s, respectively. For  $\mu \in \mathbb{R}$ ,  $\mathbb{C}^+_\mu \ := \ \{ s \ \in \ \mathbb{C} \ \mid \ \text{Re}(s) \ \geq \ \mu \} \ \ \text{and} \ \ \mathbb{C}^-_\mu \ := \ \{ s \ \in \mathbb{C} \}$  $\mathbb{C}$  | Re(s) <  $\mu$ . For positive integers k and l,  $\mathbb{R}^k$  $\mathbb{C}$  | Re(s)  $\lt \mu$ . For positive integers k and l,  $\mathbb{R}^k$  and  $\mathbb{R}^{k \times l}$  respectively denote the spaces of k-dimensional real vectors and  $k \times l$ -dimensional real matrices.  $I_k$  and<br> $r_{\text{e}}$  respectively denote the  $k \times k$  dimensional identity  $0_{k \times l}$  respectively denote the  $k \times k$ -dimensional identity<br>matrix and the  $k \times l$  dimensional zero matrix L and matrix and the  $k \times l$ -dimensional zero matrix. I and<br>matrix and zero matrices of 0 respectively denote the identity and zero matrices of appropriate dimensions. det(·), rank(·),  $\rho(\cdot)$ , and  $(\cdot)^T$ respectively denote the determinant, the rank, the spectral respectively denote the determinant, the rank, the spectral radius, and the transpose of  $(\cdot)$ . Finally, i denotes the imaginary unit imaginary unit. imaginary unit. imaginary unit. imaginary unit. respectively. For  $s \in \mathbb{C}$ , Re(s) and Im(s) denote the real and imaginary parts of s, respectively. For  $u \in \mathbb{R}$ , real and imaginary parts of s, respectively. For  $\mu \in \mathbb{R}$ ,<br> $\mathbb{C}^+$  in Eq.  $\mathbb{C}^-$  in Eq. (a) and  $\mathbb{C}^-$  in Eq. (a)  $\mathbb{C}_{\mu}^{+} := \{ s \in \mathbb{C} \mid \text{Re}(s) \geq \mu \}$  and  $\mathbb{C}_{\mu}^{-} := \{ s \in \mathbb{C} \mid \text{Re}(s) \leq \mu \}$ and  $\mathbb{R}^{\kappa \times l}$  respectively denote the spaces of k-dimensional Throughout the paper,  $\mathbb{N}$ ,  $\mathbb{R}$ , and  $\mathbb{C}$  denote the sets of non-Imoughout the paper,  $N$ ,  $M$ , and  $C$  denote the sets of non-<br>negative integers, real numbers, and complex numbers, respectively. For  $v = c$ , respectively. For  $\mu \in \mathbb{R}$ , respectively. Fear and imaginary parts of s, respectively. For  $\mu \in \mathbb{R}$ ,<br>  $\mathbb{C}_{\mu}^{+} := \{ s \in \mathbb{C} \mid \text{Re}(s) \geq \mu \}$  and  $\mathbb{C}_{\mu}^{-} := \{ s \in$  $\mathcal{L}_{\mu}$  :  $\mathcal{L}_{\beta}$  is  $\mathcal{L}_{\beta}$  integers  $\mathcal{L}_{\mu}$  integers  $\mathcal{L}_{\mu}$  and  $\mathcal{L}_{\mu}$  if  $\mathcal{L}_{\beta}$  is  $\mathcal{L}_{\beta}$  $\mathbb{C}$  | Re(s)  $\lt \mu$ . For positive integers k and l,  $\mathbb{R}^k$ and  $\kappa$  respectively denote the spaces of  $\kappa$ -dimensional real vectors and  $k \times l$ -dimensional real matrices.  $I_k$  and real vectors and  $k \times l$ -dimensional real matrices.  $I_k$  and  $0_{k \times l}$  respectively denote the  $k \times k$ -dimensional identity  $\mathcal{O}_{k \times l}$  respectively denote the  $\kappa \times \kappa$ -dimensional identity matrix and the  $k \times l$ -dimensional zero matrix. I and matrix and the  $k \times l$ -dimensional zero matrix. I and the k  $\times l$ -dimensional zero matrices of  $\sigma$  respectively denote the identity and zero matrices of appropriate dimensions. det(·), rank(·),  $\rho$ (·), and  $(\cdot)^T$  $\alpha$  appropriate dimensions. det(·), rank(·),  $\rho$ (·), and (·)<br>respectively denote the determinant, the rank, the spectral respectively denote the determinant, the rank, the spectral<br>radius, and the transpose of  $(\cdot)$ . Finally, i denotes the and  $\mathbb{R}^{k \times l}$  respectively denote the spaces of k-dimensional  $\mathcal{L}_{\mu}$  := {s ∈ C | Re(s) ∠  $\mu$ *f* and  $\mathcal{L}_{\mu}$  := {s ∈<br>  $\mathcal{L}$  | Re(s) <  $\mu$ } For positive integers k and *l*  $\mathbb{R}^k$ and  $\mathbb{R}^{k \times l}$  respectively denote the spaces of k-dimensional radius, and the transpose of (*)*. Finally, i denotes the imaginary unit.

## 2. PROBLEM STATEMENT 2. PROBLEM STATEMENT 2. PROBLEM STATEMENT 2. PROBLEM STATEMENT 2. PROBLEM STATEMENT

We consider a LTI time-delay system, to be denoted by  $\Sigma$ , described by delay-differential-algebraic equations: described by delay-differential-algebraic equations: We consider a LTI time-delay system, to be denoted by  $\Sigma$ , we consider a LTI time-delay system, to be denoted by Δ, described by delay-differential-algebraic equations: 2. PROBLEM STATEMENT<br>We consider a LTI time-delay system, to be denoted by  $\Sigma$ ,<br>described by delay-differential-algebraic equations: described by delay-differential-algebraic equations:

$$
E\dot{x}(t) = \sum_{i=0}^{\sigma} (A_i x(t - h_i) + B_i u(t - h_i))
$$
  

$$
y(t) = \sum_{i=0}^{\sigma} (C_i x(t - h_i) + D_i u(t - h_i))
$$
 (1)

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where  $x(t) \in \mathbb{R}^n$ ,  $u(t) \in \mathbb{R}^m$ , and  $y(t) \in \mathbb{R}^q$  are, respectively, the state, the input, and the output vectors at time t.  $h_1,\ldots,h_\sigma > 0$  are the time-delays, where  $\sigma$  is the number of distinct time-delays of the system.  $h_0 := 0$ is used for notational convenience. The matrices  $E, A_i, B_i$ ,  $C_i$ , and  $D_i$ ,  $i = 0, \ldots, \sigma$ , are constant real matrices. It is assumed that the matrices  $E$  and  $A_0$  satisfy

$$
rank [E A_0] = rank [E^T A_0^T]^T = n .
$$
 (2)

This assumption ensures the solvability of (1) (Hale and Verduyn-Lunel (1993)).

It is worth to emphasize that a neutral time-delay system which is described as

$$
\dot{\tilde{x}}(t) + \sum_{i=1}^{\sigma} (\tilde{E}_i \dot{\tilde{x}}(t - h_i))
$$
\n
$$
= \sum_{i=0}^{\sigma} (\tilde{A}_i \tilde{x}(t - h_i) + \tilde{B}_i u(t - h_i))
$$
\n
$$
y(t) = \sum_{i=0}^{\sigma} (\tilde{C}_i \tilde{x}(t - h_i) + \tilde{D}_i u(t - h_i))
$$
\n(3)

can be brought into the form of (1) by defining  $\delta(t)$  :=  $\tilde{x}(t) + \sum_{i=1}^{\sigma} \left( \tilde{E}_i \tilde{x}(t - h_i) \right)$  and  $x(t) := \left[ \delta(t)^T \tilde{x}(t)^T \right]^T$ .

For any given  $\epsilon \in \mathbb{R}$ , the set of  $\epsilon$ -modes of  $\Sigma$  is defined as

$$
\Omega_{\epsilon}(\Sigma) = \{ s \in \mathbb{C}_{\epsilon}^+ \mid \det(\phi(s)) = 0 \}
$$
 (4)

where  $\phi(s) := sE - \bar{A}(s)$  is the characteristic matrix of the system  $\sum_{i=0}^{\infty}$  where  $\bar{A}(s) := \sum_{i=0}^{\sigma} A_i e^{-s h_i}$ .

For any given  $\mu \in \mathbb{R}$ , where  $\mu$  is the *stability boundary*, the system  $\Sigma$  is said to be  $\mu$ -stable if there exist a  $\xi > 0$ , such that  $\Omega_{\mu-\xi}(\Sigma) = \emptyset$ . We note that, for  $\mu \leq 0$ , this definition is equivalent to exponential stability with a decay rate less than  $\mu$  (Michiels and Niculescu (2007)).

Moreover, the  $\mu$ -stability condition can also be expressed in terms of the *spectral abscissa* of the system  $\Sigma$ , which is defined as

$$
c(\Sigma) := \sup \{ \text{Re}(s) \mid \det(\phi(s)) = 0 \} .
$$
 (5)

Then,  $\Sigma$  is  $\mu$ -stable if and only if  $c(\Sigma) < \mu$ .

The spectral characteristics of a neutral time-delay system are quite complicated than a retarded time-delay system. In the stability analysis of a neutral time-delay system, the associated delay-difference equation plays an important role. Let v denote the rank deficiency of E, i.e.,  $v := n$ rank(E). Note that, when  $v = 0$  (i.e., rank(E) = n), (1) describes a retarded system. In this case, the associated delay-difference equation does not exist and  $\Omega_{\epsilon}(\Sigma)$  is a finite set for any  $\epsilon \in \mathbb{R}$ . In the case  $1 \le v \le n$ , i.e., when (1) describes a neutral system, let the unitary matrices  $U \in \mathbb{R}^{n \times v}$  and  $V \in \mathbb{R}^{n \times v}$  be such that

$$
U^T E = 0 \quad \text{and} \quad EV = 0 , \tag{6}
$$

where the columns of  $U$  and  $V$  form a minimal basis for the left and right null spaces of E, respectively. Then, due to the form of E and  $A_0$ , given in (2),  $U^T A_0 V$  is nonsingular. The associated delay-difference equation of (1) can then be expressed as

$$
\sum_{i=0}^{\sigma} \hat{A}_i x(t - h_i) = 0 , \qquad (7)
$$

where  $\hat{A}_i := U^T A_i V, i = 0, \ldots, \sigma$ . Stability of (7) is determined by the location of the roots of its characteristic equation

$$
\det(\phi_D(s)) = 0 , \qquad (8)
$$

where

$$
\phi_D(s) := \sum_{i=0}^{\sigma} \hat{A}_i e^{-sh_i} . \tag{9}
$$

In this case,  $(7)$  is  $\mu$ -stable if and only if all the infinitely many roots of (8) are located to the left hand side of the  $\mu-\xi$  axis, for some  $\xi>0$ . We can also express the stability condition in terms of the spectral abscissa of (7), which is defined as,

$$
c_D(\Sigma) := \sup \{ \text{Re}(s) \mid \det(\phi_D(s)) = 0 \} . \tag{10}
$$

Then, (7) is  $\mu$ -stable if and only if  $c_D(\Sigma) < \mu$ . In fact,  $\mu$ stability of  $(7)$  is a necessary condition for the  $\mu$ -stability of  $\Sigma$ .

Although (10) is continuous in the entries of the system matrices, it is not continuous in the time-delays. As a consequence of this, the high frequency roots of the delaydifference equation, accordingly  $c_D(\Sigma)$ , may be highly sensitive to infinitesimal perturbations in the time-delays. This situation was the main motivation of Hale and Verduyn-Lunel (1993) to introduce the concept of strong stability.  $\Sigma$  is said to be strongly  $\mu$ -stable if it is  $\mu$ stable and remains  $\mu$ -stable for small changes in the timedelays. Furthermore,  $(7)$  is strongly  $\mu$ -stable if and only if  $\gamma_\mu(\Sigma) < 1$ , where

$$
\gamma_{\mu}(\Sigma) := \max_{\theta \in [0, 2\pi]^{\sigma}} \rho \left( \sum_{k=1}^{\sigma} \hat{A}_0^{-1} \hat{A}_k e^{-\mu h_k} e^{i\theta_k} \right),
$$

where  $\theta := {\theta_1, \ldots, \theta_{\sigma}}$  (see, e.g., Michiels (2011) for the case  $\mu = 0$ ; the general case follows by the transformation  $s \rightarrow s - \mu$ ). Although the above condition is enough to decide on the strong  $\mu$ -stability of (7), it is still useful to know  $c_D(\Sigma)$  since it gives direct information about the location of the roots of (8). Considering the hypersensitivity of  $c_D(\Sigma)$ , a so-called *safe* upper bound which is robust to the infinitesimal changes in the timedelays must be introduced. The safe upper bound, which will be indicated as  $C_D(\Sigma)$ , is equal to the unique root of  $g(\zeta) = 1$ , where

$$
g(\zeta):=\max_{\theta\,\in\,[0,\,2\pi]^\sigma}\,\,\rho\left(\sum_{k=1}^\sigma \hat A_0^{-1}\hat A_ke^{-\zeta h_k}e^{\mathrm{i}\theta_k}\right)\,.
$$

Hence, the delay-difference equation (7) is strongly  $\mu$ stable if and only if  $C_D(\Sigma) < \mu$ .

The strong  $\mu$ -stability condition for  $\Sigma$ , then, can be expressed in terms of both  $\gamma_{\mu}(\Sigma)$  and  $C_D(\Sigma)$ .  $\Sigma$  is strongly  $\mu$ -stable if and only if  $c(\Sigma) < \mu$  and  $C_D(\Sigma) < \mu$ , while the latter condition can also be expressed as  $\gamma_\mu(\Sigma) < 1$ .

In this work, to strongly stabilize a system of the form (1), we consider finite-dimensional LTI output feedback controllers of the form

$$
\dot{z}(t) = Fz(t) + Gy(t)
$$
  
\n
$$
u(t) = Hz(t) + Ky(t)
$$
\n(11)

where  $z(t) \in \mathbb{R}^l$  is the state of the controller at time t and  $F, G, H$ , and  $K$  are real constant matrices. Here, the Download English Version:

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