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Controller Design for Neutral Time-Delay Systems by Nonsmooth Optimization ⋆

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Abstract: Strong stabilization of linear time-invariant neutral time-delay systems is considered. A controller design algorithm based on a nonsmooth optimization approach is proposed. An initialization procedure for the non-convex optimization problem is also presented. Finally, the proposed approach is demonstrated by an example.

Keywords: Time-delay systems; neutral systems; strong stabilization; nonsmooth optimization.

1. INTRODUCTION

Eigenvalue-based methods have recently become popular in the stabilization of both neutral and retarded linear time-invariant (LTI) time-delay systems (e.g., see Michiels and Niculescu (2007) and references therein). Stabilization of a neutral time-delay system, however, is much harder than that of a retarded time-delay system. There are two main reasons for that. First of all, although a retarded LTI time-delay system has only a finite number of modes in any given right-half complex plane, this is not true for a neutral system (Niculescu (2001)). Secondly, although the modes of a retarded LTI time-delay system varies continuously with system parameters and the time-delays, the high-frequency modes of a neutral LTI time-delay system are sensitive to infinitesimal changes in the time-delays. To overcome the latter difficulty, the concept of *strong stability* was introduced by Hale and Verduyn-Lunel (1993). A time-delay system is said to be strongly stable if it is stable and remains stable under infinitesimal changes in the time-delays. One of the eigenvalue-based stabilization methods proposed for LTI time-delay systems is the so-called *continuous pole placement* method, which was originally proposed by Michiels et al. (2002) for retarded time-delay systems. This approach was then extended to neutral time-delay systems by Michiels and Vyhlidal (2005), where strong stability was considered. This method has also been extended for decentralized controller design by Erol and İftar (2014, 2015). As an alternative to the continuous pole placement method, a *nonsmooth optimization*-based method was proposed by Vanbiervliet et al. (2008) for retarded time-delay systems. This method was then employed by Vyhlidal et al. (2011) to design strongly stabilizing state-derivative controllers for retarded time-delay systems. Although, the given systems were assumed to be retarded in Vyhlidal et al. (2011), the closed-loop system becomes neutral due to the structure of the controller used. Finally, strong stabilization of neutral time-delay systems, written in the form of delay-differential-algebraic equations, was considered by Michiels (2011). It should, however, be noted that since the optimization problem is not convex, the choice of the initial parameters plays an important role in the success of those algorithms. In the present work, we employ the nonsmooth optimization approach to strongly stabilize a given LTI neutral time-delay system along the lines of Michiels (2011). As a novel contribution, we propose an initialization procedure for the optimization problem. Furthermore, as in Özer and İftar (2015), where decentralized controller design by nonsmooth optimization for retarded time-delay systems was considered, we structure the controller matrices in certain canonical forms. The problem is stated in Section 2 and its proposed solution is given in Section 3. An example is presented in Section 4 and some concluding remarks are given in Section 5.

Throughout the paper, $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{C}$ denote the sets of non-negative integers, real numbers, and complex numbers, respectively. For $s \in \mathbb{C}$, $\mathrm{Re}(s)$ and $\mathrm{Im}(s)$ denote the real and imaginary parts of s, respectively. For $\mu \in \mathbb{R}$, $\mathbb{C}^+_\mu := \{s \in \mathbb{C} \mid \mathrm{Re}(s) \geq \mu\}$ and $\mathbb{C}^-_\mu := \{s \in \mathbb{C} \mid \mathrm{Re}(s) < \mu\}$. For positive integers k and l, $\mathbb{R}^k$ and $\mathbb{R}^{k\times l}$ respectively denote the spaces of k-dimensional real vectors and $k \times l$-dimensional real matrices. I_k and $0_{k\times l}$ respectively denote the $k \times k$-dimensional identity matrix and the $k \times l$-dimensional zero matrix. I and 0 respectively denote the identity and zero matrices of appropriate dimensions. $\det(\cdot)$, $\mathrm{rank}(\cdot)$, $\rho(\cdot)$, and $(\cdot)^T$ respectively denote the determinant, the rank, the spectral radius, and the transpose of $(\cdot)$. Finally, i denotes the imaginary unit.

2. PROBLEM STATEMENT

We consider a LTI time-delay system, to be denoted by Σ, described by delay-differential-algebraic equations:

$$\begin{aligned} E\dot{x}(t) &= \sum_{i=0}^{\sigma} \big(A_i x(t-h_i) + B_i u(t-h_i)\big) \\ y(t) &= \sum_{i=0}^{\sigma} \big(C_i x(t-h_i) + D_i u(t-h_i)\big) \end{aligned} \qquad (1)$$

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where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^q$ are, respectively, the state, the input, and the output vectors at time t. $h_1, \ldots, h_\sigma > 0$ are the time-delays, where σ is the number of distinct time-delays of the system. $h_0 := 0$ is used for notational convenience. The matrices E, A_i, B_i, C_i, and D_i, $i = 0, \ldots, \sigma$, are constant real matrices. It is assumed that the matrices E and A_0 satisfy

$$\mathrm{rank}\,[E\ A_0] = \mathrm{rank}\left[E^T\ A_0^T\right]^T = n\ . \tag{2}$$

This assumption ensures the solvability of (1) (Hale and Verduyn-Lunel (1993)).

It is worth to emphasize that a neutral time-delay system which is described as

$$\begin{aligned} \dot{\tilde{x}}(t) + \sum_{i=1}^{\sigma} \left(\tilde{E}_i \dot{\tilde{x}}(t-h_i)\right) &= \sum_{i=0}^{\sigma} \left(\tilde{A}_i \tilde{x}(t-h_i) + \tilde{B}_i u(t-h_i)\right) \\ y(t) &= \sum_{i=0}^{\sigma} \left(\tilde{C}_i \tilde{x}(t-h_i) + \tilde{D}_i u(t-h_i)\right) \end{aligned} \tag{3}$$

can be brought into the form of (1) by defining $\delta(t) := \tilde{x}(t) + \sum_{i=1}^{\sigma} \left(\tilde{E}_i \tilde{x}(t-h_i)\right)$ and $x(t) := \left[\ \delta(t)^T\ \ \tilde{x}(t)^T\ \right]^T$.

For any given $\epsilon \in \mathbb{R}$, the set of ϵ-modes of Σ is defined as

$$\Omega_\epsilon(\Sigma) = \{s \in \mathbb{C}_\epsilon^+ \mid \det\left(\phi(s)\right) = 0\} \tag{4}$$

where $\phi(s) := sE - \bar{A}(s)$ is the characteristic matrix of the system Σ, where $\bar{A}(s) := \sum_{i=0}^{\sigma} A_i e^{-sh_i}$.

For any given $\mu \in \mathbb{R}$, where μ is the *stability boundary*, the system Σ is said to be *μ-stable* if there exist a $\xi > 0$, such that $\Omega_{\mu-\xi}(\Sigma) = \emptyset$. We note that, for $\mu \leq 0$, this definition is equivalent to exponential stability with a decay rate less than μ (Michiels and Niculescu (2007)).

Moreover, the μ-stability condition can also be expressed in terms of the *spectral abscissa* of the system Σ, which is defined as

$$c(\Sigma) := \sup\left\{\mathrm{Re}(s) \mid \det\left(\phi(s)\right) = 0\right\}\ . \tag{5}$$

Then, Σ is *μ-stable* if and only if $c(\Sigma) < \mu$.

The spectral characteristics of a neutral time-delay system are quite complicated than a retarded time-delay system. In the stability analysis of a neutral time-delay system, the associated delay-difference equation plays an important role. Let v denote the rank deficiency of E, i.e., $v := n - \mathrm{rank}(E)$. Note that, when $v = 0$ (i.e., $\mathrm{rank}(E) = n$), (1) describes a retarded system. In this case, the associated delay-difference equation does not exist and $\Omega_\epsilon(\Sigma)$ is a finite set for any $\epsilon \in \mathbb{R}$. In the case $1 \leq v \leq n$, i.e., when (1) describes a neutral system, let the unitary matrices $U \in \mathbb{R}^{n\times v}$ and $V \in \mathbb{R}^{n\times v}$ be such that

$$U^T E = 0 \quad \text{and} \quad EV = 0\ , \tag{6}$$

where the columns of U and V form a minimal basis for the left and right null spaces of E, respectively. Then, due to the form of E and A_0, given in (2), $U^T A_0 V$ is nonsingular. The associated delay-difference equation of (1) can then be expressed as

$$\sum_{i=0}^{\sigma} \hat{A}_i x(t-h_i) = 0\ , \tag{7}$$

where $\hat{A}_i := U^T A_i V$, $i = 0, \ldots, \sigma$. Stability of (7) is determined by the location of the roots of its characteristic equation

$$\det(\phi_D(s)) = 0\ , \tag{8}$$

where

$$\phi_D(s) := \sum_{i=0}^{\sigma} \hat{A}_i e^{-sh_i}\ . \tag{9}$$

In this case, (7) is μ-stable if and only if all the infinitely many roots of (8) are located to the left hand side of the $\mu - \xi$ axis, for some $\xi > 0$. We can also express the stability condition in terms of the spectral abscissa of (7), which is defined as,

$$c_D(\Sigma) := \sup\left\{\mathrm{Re}(s) \mid \det\left(\phi_D(s)\right) = 0\right\}\ . \tag{10}$$

Then, (7) is μ-stable if and only if $c_D(\Sigma) < \mu$. In fact, μ-stability of (7) is a necessary condition for the μ-stability of Σ.

Although (10) is continuous in the entries of the system matrices, it is not continuous in the time-delays. As a consequence of this, the high frequency roots of the delay-difference equation, accordingly $c_D(\Sigma)$, may be highly sensitive to infinitesimal perturbations in the time-delays. This situation was the main motivation of Hale and Verduyn-Lunel (1993) to introduce the concept of *strong stability*. Σ is said to be *strongly μ-stable* if it is μ-stable and remains μ-stable for small changes in the time-delays. Furthermore, (7) is strongly μ-stable if and only if $\gamma_\mu(\Sigma) < 1$, where

$$\gamma_\mu(\Sigma) := \max_{\theta\,\in\,[0,\,2\pi]^\sigma} \rho\left(\sum_{k=1}^{\sigma} \hat{A}_0^{-1} \hat{A}_k e^{-\mu h_k} e^{\mathrm{i}\theta_k}\right),$$

where $\theta := \{\theta_1, \ldots, \theta_\sigma\}$ (see, e.g., Michiels (2011) for the case $\mu = 0$; the general case follows by the transformation $s \to s - \mu$). Although the above condition is enough to decide on the strong μ-stability of (7), it is still useful to know $c_D(\Sigma)$ since it gives direct information about the location of the roots of (8). Considering the hypersensitivity of $c_D(\Sigma)$, a so-called *safe* upper bound which is robust to the infinitesimal changes in the time-delays must be introduced. The safe upper bound, which will be indicated as $C_D(\Sigma)$, is equal to the unique root of $g(\zeta) = 1$, where

$$g(\zeta) := \max_{\theta\,\in\,[0,\,2\pi]^\sigma} \rho\left(\sum_{k=1}^{\sigma} \hat{A}_0^{-1} \hat{A}_k e^{-\zeta h_k} e^{\mathrm{i}\theta_k}\right).$$

Hence, the delay-difference equation (7) is strongly μ-stable if and only if $C_D(\Sigma) < \mu$.

The strong μ-stability condition for Σ, then, can be expressed in terms of both $\gamma_\mu(\Sigma)$ and $C_D(\Sigma)$. Σ is strongly μ-stable if and only if $c(\Sigma) < \mu$ and $C_D(\Sigma) < \mu$, while the latter condition can also be expressed as $\gamma_\mu(\Sigma) < 1$.

In this work, to strongly stabilize a system of the form (1), we consider finite-dimensional LTI output feedback controllers of the form

$$\begin{aligned} \dot{z}(t) &= Fz(t) + Gy(t) \\ u(t) &= Hz(t) + Ky(t) \end{aligned}, \tag{11}$$

where $z(t) \in \mathbb{R}^l$ is the state of the controller at time t and F, G, H, and K are real constant matrices. Here, the