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Strong Stabilization of Lossless Propagation Time-Delay Systems by Continuous Pole Placement ⋆

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Abstract: Stabilizing controller design problem for a class of LTI neutral time-delay systems, namely *lossless propagation* time-delay systems, is considered. Effect of small time-delay perturbations on the stability is also taken into account. A dynamic output feedback controller design approach, based on the continuous pole placement algorithm, is proposed. A numerical example is also presented to demonstrate the proposed approach.

Keywords: Time-delay systems; neutral systems; feedback stabilization; pole placement.

1. INTRODUCTION

Many examples of neutral time-delay systems can be found in nature or in man-made systems (e.g., see Slemrod (1971), Răsvan (1974), Niculescu and Brogliato (1999), Bellen et al. (1999)). An important class of neutral time-delay systems is the so-called *lossless propagation* (Niculescu (2001)) time-delay systems, whose dynamics can be described by coupled delay-differential-algebraic equations. Stabilization of neutral time-delay systems are, in general, more difficult than that of retarded time-delay systems. Furthermore, unlike a retarded time-delay system, a neutral time-delay system may become unstable by small perturbations in the time-delays. Because of this, the concept of *strong stability* was introduced by Hale and Verduyn-Lunel (1993). A time-delay system is said to be *strongly stable* if it is stable and remains stable despite infinitesimal changes in the time-delays.

One method, which can be employed in the stabilization of linear time-invariant (LTI) time-delay systems, is the *continuous pole placement algorithm*, which was originally proposed for retarded time-delay systems by Michiels et al. (2002) and extended for neutral time-delay systems by Michiels and Vyhlidal (2005). Although the algorithm was originally proposed for static state vector feedback, it was extended to dynamic output feedback by Erol and İftar (2014, 2015), where only retarded time-delay systems were considered. In the present work, we consider a quite general class of LTI neutral time-delay systems, namely *lossless propagation* time-delay systems. We propose a controller design algorithm, based on the continuous pole placement algorithm, to strongly stabilize such systems by finite-dimensional dynamic output feedback controllers. The problem is stated in Section 2 and its proposed solution is given in Section 3. Section 4 includes an example to demonstrate the proposed approach.

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Throughout the paper, $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{N}$, denote the sets of, respectively, complex numbers, real numbers and non-negative integers. For $s \in \mathbb{C}$, $\mathrm{Re}(s)$ denotes the real part of s. For $a, b \in \mathbb{R}$, with $a < b$, (a, b) and $[a, b]$ indicate, respectively, the open and the closed intervals of the real line between a and b. For $k, l \in \mathbb{N}$, $\mathcal{F}^k$ and $\mathcal{F}^{k\times l}$ denote the spaces of, respectively, k-dimensional vectors and $k \times l$-dimensional matrices with elements in $\mathcal{F}$, where $\mathcal{F}$ is either $\mathbb{R}$ or $\mathbb{C}$. I_k and $0_{k\times l}$ respectively denote the $k \times k$-dimensional identity and the $k \times l$-dimensional zero matrices. When the dimensions are apparent, we use I and 0 to denote respectively the identity and the zero matrices. For $\mu \in \mathbb{R}$, $\mathbb{C}^-_\mu := \{s \in \mathbb{C} \mid \mathrm{Re}(s) < \mu\}$ and $\mathbb{C}^+_\mu := \{s \in \mathbb{C} \mid \mathrm{Re}(s) \geq \mu\}$. $\det(\cdot)$, $\mathrm{rank}(\cdot)$, $\rho(\cdot)$, $\|\cdot\|$, $(\cdot)^\dagger$, $(\cdot)^T$, and $(\cdot)^*$ respectively denote the determinant, the rank, the spectral radius, the 2-norm, the Moore-Penrose generalized inverse, the transpose, and the complex-conjugate transpose of $(\cdot)$. Finally, i denotes the imaginary unit.

2. PROBLEM STATEMENT

Consider a LTI time-delay system, Σ, described as

$$\begin{aligned} \sum_{i=0}^{\sigma} \left(E_i \dot{x}(t-h_i)\right) &= \sum_{i=0}^{\sigma} \left(A_i x(t-h_i) + B_i u(t-h_i) \right), \\ y(t) &= \sum_{i=0}^{\sigma} \left(C_i x(t-h_i) + D_i u(t-h_i) \right) \end{aligned} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector at time t, and $u(t) \in \mathbb{R}^p$ and $y(t) \in \mathbb{R}^q$ are, respectively, the input and the output vectors at time t. The matrices E_i, A_i, B_i, C_i, and D_i $(i = 0, \ldots, \sigma)$ are constant real matrices, $h_0 := 0$ is defined for notational convenience, $h_i > 0$, $i = 1, \ldots, \sigma$ are the time-delays, and σ is the number of distinct time-delays involved. Here, we restrict ourselves to *lossless propagation* time-delay systems (Niculescu (2001)), which can be represented by (1), where (by an appropriate state transformation) E_i and A_i matrices $(i = 0, \ldots, \sigma)$ can be written as

$$E_i = \begin{bmatrix} E_i^{11} & 0 \\ 0 & 0_{n_2\times n_2} \end{bmatrix} \quad \text{and} \quad A_i = \begin{bmatrix} A_i^{11} & A_i^{12} \\ A_i^{21} & A_i^{22} \end{bmatrix}, \quad (2)$$

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where E_i^{11} and A_i^{11} are $n_1 \times n_1$ dimensional and A_i^{22} are $n_2 \times n_2$ dimensional matrices with $\text{rank}(E_0^{11}) = n_1$ and $\text{rank}(A_0^{22}) = n_2$, where $n_1, n_2 \in \mathbb{N}$ and $n_1 + n_2 = n$.

Relating to the system Σ, described by (1), let us first present the following definitions, which are borrowed from Erol and İftar (2016).

Definition 1. For any given $\mu \in \mathbb{R}$, the set of *μ-modes* of the system Σ, described by (1), is defined as

$$\Omega_\mu(\Sigma) := \left\{ s \in \mathbb{C}_\mu^+ \mid \phi_\Sigma(s) = 0 \right\} , \tag{3}$$

where $\phi_\Sigma(s) := \det\left(s\bar{E}(s) - \bar{A}(s)\right)$ is the *characteristic function* of the system Σ, where

$$\bar{E}(s) := \sum_{i=0}^{\sigma} E_i e^{-sh_i} \quad \text{and} \quad \bar{A}(s) := \sum_{i=0}^{\sigma} A_i e^{-sh_i} . \tag{4}$$

Furthermore, any $s_o \in \mathbb{C}$ for which $\phi_\Sigma(s_o) = 0$ is said to be a *simple mode* of Σ, if $\frac{d\phi_\Sigma(s)}{ds}|_{s=s_o} \neq 0$, and is said to be a *multiple mode* of Σ, otherwise. Moreover, $k \in \mathbb{N}$ is said to be the *multiplicity* of s_o, if $\frac{d^r\phi_\Sigma(s)}{ds^r}|_{s=s_o} = 0$, for $r = 1, \ldots, k-1$, but $\frac{d^k\phi_\Sigma(s)}{ds^k}|_{s=s_o} \neq 0$.

Definition 2. For any given $\mu \in \mathbb{R}$, the system Σ is said to be *μ-stable* if $\Omega_{\mu-\delta}(\Sigma) = \emptyset$ for some $\delta > 0$. Furthermore, a controller $\mathcal{K}$ is said to *μ-stabilize* the system Σ, if the closed-loop system obtained by applying the controller $\mathcal{K}$ to system Σ is μ-stable.

Remark 1. For any $\mu \leq 0$, μ-stability of Σ, as defined above, is equivalent to its exponential stability with decay rate less than μ (Michiels and Niculescu (2007)).

Although the system Σ has infinitely many modes, in general, it is known that (Niculescu (2001)) $\Omega_\mu(\Sigma)$ is a finite set for any $\mu > \mu_f(\Sigma)$, where $\mu_f(\Sigma) := \max(\mu_E, \mu_A)$, where

$$\mu_E := \sup\left\{ \text{Re}(s) \;\middle|\; \det\left(\sum_{i=0}^{\sigma} E_i^{11} e^{-sh_i}\right) = 0 \right\} \tag{5}$$

and

$$\mu_A := \sup\left\{ \text{Re}(s) \;\middle|\; \det\left(\sum_{i=0}^{\sigma} A_i^{22} e^{-sh_i}\right) = 0 \right\} . \tag{6}$$

Although the modes of Σ with finite magnitute change continuously with respect to both entries of E_i and A_i matrices and the time-delays, it is known that both μ_E and μ_A are sensitive to infinitesimal changes in the time-delays (Hale and Verduyn-Lunel (1993)). Because of this, the concept of *strong stability* was introduced by Hale and Verduyn-Lunel (1993).

Definition 3. For any given $\mu \in \mathbb{R}$, the system Σ is said to be *strongly μ-stable* if it is μ-stable and remains μ-stable when subjected to infinitesimal changes in the time-delays.

Let X denote either E or A and $\mu_X^{\boldsymbol{\delta}}$ denote the supremum of μ_X over all perturbations of the time-delays h_i, $i = 1, \ldots, \sigma$, within the interval $(\max(0, h_i - \delta_i), h_i + \delta_i)$, where $\delta_i > 0$ and $\boldsymbol{\delta} := [\,\delta_1 \cdots \delta_\sigma\,]$. Then, define $\mu_X^s := \lim_{\boldsymbol{\delta}\to 0} \mu_X^{\boldsymbol{\delta}}$ and $\mu_f^s(\Sigma) := \max(\mu_E^s, \mu_A^s)$. Then, Σ is strongly μ-stable if and only if $\mu_f^s(\Sigma) < \mu$ and $\Omega_\mu(\Sigma) = \emptyset$. Therefore, to strongly μ-stabilize Σ, we have to find a controller which achieves $\mu_f^s(\Sigma^c) < \mu$ and $\Omega_\mu(\Sigma^c) = \emptyset$, where Σ^c denotes the closed-loop system. It is known that μ_E^s can not be changed by a proper controller (Loiseau et al. (2002)). Although it is possible to change μ_A^s, in this work, for some $\epsilon > 0$, we will assume that the given system satisfies both $\mu_E^s < \mu - \epsilon$ and $\mu_A^s < \mu - \epsilon$, i.e., $\mu_f^s(\Sigma) < \mu - \epsilon$. Here, we require $\mu_f^s(\Sigma) < \mu - \epsilon$, rather than $\mu_f^s(\Sigma) < \mu$, so that we can compute $\Omega_{\mu-\epsilon}(\Sigma)$. Therefore, we will only try to move finitely many modes in $\Omega_\mu(\Sigma)$ towards $\mathbb{C}_\mu^-$. To achieve this, however, the given system must be *μ-stabilizable* and *μ-detectable* (Richard (2003)).

Definition 4. For any given $\mu \in \mathbb{R}$, the system Σ is said to be *μ-stabilizable* if

$$\text{rank}\left[\, s\bar{E}(s) - \bar{A}(s) \;\; \bar{B}(s) \,\right] = n , \tag{7}$$

and is said to be *μ-detectable* if

$$\text{rank}\begin{bmatrix} s\bar{E}(s) - \bar{A}(s) \\ \bar{C}(s) \end{bmatrix} = n \tag{8}$$

for all $s \in \Omega_{\mu-\delta}(\Sigma)$ for some $\delta > 0$, where $\bar{A}(s)$ and $\bar{E}(s)$ are as defined in (4), and

$$\bar{B}(s) := \sum_{i=0}^{\sigma} B_i e^{-sh_i} \quad \text{and} \quad \bar{C}(s) := \sum_{i=0}^{\sigma} C_i e^{-sh_i} . \tag{9}$$

Therefore, in the rest of the paper, we assume that the given system Σ is $(\mu-\epsilon)$-stabilizable and $(\mu-\epsilon)$-detectable, where ϵ is as introduced in the paragraph preceding Definition 4.

To strongly stabilize the system Σ, we consider finite-dimensional dynamic output feedback controllers of the form:

$$\begin{array}{l} \dot{z}(t) = Fz(t) + Gy(t) \\ u(t) = Hz(t) + Ky(t) \end{array} , \tag{10}$$

where $z(t) \in \mathbb{R}^m$ is the state vector of the controller at time t, where $m \in \mathbb{N}$ is the controller dimension. Since the input-output behaviour of the controller (10) is unique up to a similarity transformation, in order to minimize the number of free parameters of the controller, we assume that $F \in \mathbb{R}^{m\times m}$, $G \in \mathbb{R}^{m\times q}$, $H \in \mathbb{R}^{p\times m}$, and $K \in \mathbb{R}^{p\times q}$ are structured in a multivariable canonical form. The forms we use are given in Erol and İftar (2015). In any of these forms, the number of the free controller parameters is

$$\hat{m} := m(p + q) + pq . \tag{11}$$

We note that, such a structuring reduces the number of free parameters by m^2. We also note that, when $m = 0$, the controller (10) reduces to a static output feedback controller of the form: $u(t) = Ky(t)$, in which case $\hat{m} = pq$ and there is no structuring.

Now, let B_i, C_i, $i = 0, \ldots, \sigma$, and $x(t)$ be partitioned as $B_i = \left[\, (B_i^1)^T \; (B_i^2)^T \,\right]^T$, $C_i = \left[\, C_i^1 \; C_i^2 \,\right]$, and $x(t) = \left[\, x_1^T(t) \; x_2^T(t) \,\right]^T$, where the partitioning is compatible with the partitioning in (2). Also let $v(t) := \sum_{i=0}^{\sigma} E_i^{11} x_1(t - h_i)$ and

$$\eta(t) := \left[\, v^T(t) \; z^T(t) \; x^T(t) \; y^T(t) \; u^T(t) \,\right]^T \in \mathbb{R}^{\hat{n}}, \tag{12}$$

where $\hat{n} := n_1 + m + n + q + p$. Then, the closed-loop system dynamics, obtained by applying controller (10) to Σ, can be described as

$$\hat{E}\dot{\eta}(t) = \sum_{i=0}^{\sigma} \hat{A}_i \eta(t - h_i) , \tag{13}$$

where $\hat{E} := \begin{bmatrix} I_{n_1+m} & 0 \\ 0 & 0_{(n+q+p)\times(n+q+p)} \end{bmatrix}$,