Properties of a Robust Kalman Filter ⋆

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Abstract: Almost all the communication and control systems may suffer from insufficient measurement data either due to sensor faults or communication errors. Standard Kalman filter predicts state of the system and then tunes that with the help of newly arrived observations. But in the case of insufficient data the question arises as how to compensate the loss of observation in the state estimation. In this paper a robust estimation design is presented for a sampled linear system where the sensor readings are subjected to random loss. Several easy-to-implement approaches are discussed in this paper. A brief description is stated on the design structure, affected state and minimum error covariance matrix. A comprehensive comparison survey for these approaches is presented which shows various features like computational time, innovation, convergence of the affected riccati equation, etc.

Keywords: Kalman filter, Loss of observations, Innovation, Open loop process.

1. INTRODUCTION

1.1 Nomenclature

Abbreviations and notations are defined which will be used extensively in the paper.

$LOOB$	Loss of Observations
OL	Open loop (also will be known as Cat-A approach)
$Cat-BApp$	Category-B approaches
z_k	Actual observation vector
$\hat{z}_k$	Calculated observation vector (Filtered Response)
$\bar{z}_k$	Observation vector in LOOB case
$\bar{K}$	Kalman gain calculated in LOOB case
$P_k^{\{1\}}$	Predicted Error Covariance Matrix in the LOOB case
$P_k^{\{2\}}$	Filtered Error Covariance Matrix in the LOOB case
$x_k^{\{1\}}$	Predicted state estimate in LOOB case
$x_k^{\{2\}}$	Filtered state estimate in LOOB case

We would call all the vectors and matrices in the loss of observations case as approximated, e.g. $P_k^{\{2\}}$ as approximated filtered error covariance at time step k.

1.2 Problem Statement

Since the landmark research by Kalman [1960], Kalman filter has been extensively used in numerous research areas and applications. R. Kalman presented a recursive solution for state estimation for a discrete time LTI system. State estimation is one of the key research area in both control and communication networks. With new emerging technologies e.g. scale integration and microelectromechanical system, control and communication networks are coupling together, Shi et al. [2005]. But loss of information is a non-trivial case of study in both control and communication systems. This loss may be due to faulty sensors, limited bandwidth of communication channels, confined memory space, mismatching of measurement instruments etc. And this might be the reason that loss of observation in control and communication systems remains a very hot research topic for researchers during the last decade. Kalman filter, being a versatile tool for estimation of states and parameters, could face a situation where data may not be available for measurement update step. Sinopoli and Schenato [2007], Micheli [2001], Schenato [2005] and Liu and Goldsmith [2004], have studied LOOB, while running the Kalman filter in a open loop fashion, i.e. whenever observation is lost, the predicted quantities are processed for next iteration, without any update.

In Sinopoli and Schenato [2007], the authors have discussed the affected stability of the state estimation and shown a threshold limit of data loss, above which the expected value of error covariance becomes unbounded as the time goes to infinity. Observations from different sensors are treated collectively, contrary to Liu and Goldsmith [2004], where different sensor readings are treated individually; so observations may be fully received, partially received or fully lost. In both of these papers, lower and upper bounds of the threshold value for the loss have been provided.

In Micheli [2001], delay in the data arrival is considered. In Schenato [2005], a system might be subjected to both LOOB and delay of observation at the same time. In all these papers except ?], suggested designs of Kalman filter, jump between an OL estimator when there is LOOB and a closed loop estimator when the observation arrives at destination. And hence, designed estimator is strongly time-varying and stochastic in nature.

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In order to avoid random sampling and stochastic behaviour of the designed Kalman filter, Khan and Gu [2009], has proposed a few approaches to compensate the loss of observations in the state estimation. Except the first approach all of them are based on previous observations. The authors have described all the possible merits and demerits for those approaches. In this paper we review and explore some analytic study for those approaches.
This paper is organized as: Section-2 describes the basic Kalman filter design and algorithm which would be helpful in understanding the LOOB algorithms. Section-3 presents the proposed approaches in various forms along with the algorithms. Section-4 is the main theme of this paper which shows all the possible parameters which might be affected from the LOOB along with the necessary discussion for these proposed approaches. Section-5 describes the example, simulated for evaluating these proposed approaches and their various features. And in the last section the conclusion is presented.

2. BASIC STRUCTURE OF KALMAN FILTER

2.1 Plant Model

Consider the following discrete time LTI system

$$x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1} \tag{1}$$

$$z_k = Cx_k + v_k \tag{2}$$

where $k \in \Re = \{0, 1, 2, ...\}$, $x,w \in \Re^n$, $u \in \Re^l$, $z \in \Re^m$, $A \in \Re^{n*n}$ is the state transition matrix, $B \in \Re^{n*l}$ is the input matrix, $C \in \Re^{m*n}$ output matrix and (x_0, w_k, v_k) are Gaussian, uncorrelated, white noise sequences with mean $(\bar{x}_0, 0, 0)$ and covariance (P_0, Q_k, R_k) respectively.

2.2 Kalman filter

Priori Step: This step is based on the system model. Predicted state and the corresponding error covariance are,

$$x_{k|k-1} = Ax_{k-1|k-1} + Bu_{k-1} \tag{3}$$

$$P_{k|k-1} = AP_{k-1|k-1}A^T + Q_{k-1} \tag{4}$$

The error (and thereafter, error covariance) can be derived by substituting (1) and (3) in

$$e_{k|k-1} = x_k - x_{k|k-1} \tag{5}$$

Kalman Gain: The optimal value of the Kalman gain matrix is defined as,

$$K_k = P_{k|k-1}C^T[CP_{k|k-1}C^T + R_k]^{-1} \tag{6}$$

The predicted entities are then updated with the help of observation and Kalman gain as,

Posteriori Step:

$$x_{k|k} = x_{k|k-1} + K_k(z_k - Cx_{k|k-1}) \tag{7}$$

$$\begin{aligned} P_{k|k} &= P_{k|k-1} - K_kCP_{k|k-1} \\ &= (I - K_kC)P_{k|k-1} \end{aligned} \tag{8}$$

The above five equations (3)-(8), excluding (5), describe the structure of the Kalman filter. The basic Kalman filtering can be summarized in the following algorithm as this would be helpful in the OL estimation algorithm and Cat-B estimation algorithm. Basic Kalman filter can be analyzed with respect to various features, like innovation vector and matrix, measurement update step, covariance matrices and convergence to a constant solution etc.

Algorithm 1 :Basic Kalman Filter Algorithm

1: At time step: $k - 1$,
   Prediction is carried out as
   $x_{k|k-1} = Ax_{k-1|k-1} + Bu_{k-1}$: State Estimate
   $P_{k|k-1} = AP_{k|k}A^T + Q_k$: Error Covariance Matrix
2: Time-step is updated
3: $z_k (= Cx_k + v_k)$: Observation arrives
4: Calculate $(z_k - Cx_{k|k-1})$: Innovation (or residual) vector
5: Calculate $(CP_{k|k-1}C^T + R_k)$: Innovation matrix
6: Calculate $K_k = P_{k|k-1}C^T(CP_{k|k-1}C^T + R_k)^{-1}$: Kalman Gain matrix
7: Measurement update step :
   $x_{k|k} = x_{k|k-1} + K_k(z_k - Cx_{k|k-1})$: Updated State and
   $P_{k|k} = (I - K_kC)P_{k|k-1}$: Updated Error Covariance Matrix
8: Return to step(1) i.e. Prediction;

3. APPROACHES FOR LOOB

We consider the case when in equation (7) (step-4 in the basic Kalman filter algorithm), the observation vector (z) is not available. To compensate this unavailability is the main concern of this paper. We discuss several approaches to offset this unavailability. It can be seen that the designed filter is still converging to the same constant solution, once the observations are available again. The other features of basic Kalman filter will also be discussed. The approaches are classified in two categories. Category-A consists of only one approach known as Open Loop (OL), which has been studied in the papers mentioned earlier. The reason it has been place alone in Cat-A is that, it does not depend on the past data. Category-B consists of four approaches, all of which are based on previous observations. Although in Cat-B, many other approaches could be introduced, at the moment only four of them are sufficient to provide the basic concept of utilizing the previous observations. These approaches have various advantages and disadvantages which will be explained shortly.
For simplicity, define a variable γ_k such that

$$\gamma_k = \begin{cases} 1, \text{ if there is no LOOB at time step k} \\ 0, \text{ otherwise} \end{cases} \tag{9}$$

In OL technique whenever data is lost (i.e. $\gamma_k = 0$), no filtration (posteriori step) is performed, contrarily Cat-B approaches utilize previous observations to form an approximation to carry out the measurement update step. The approximated observation vector $\bar{z}$ in Cat-B approaches can be represented in a unified form:

$$\bar{z}_{k+1} = \sum_{i=0}^{p} \alpha_i z_{k-i} \tag{10}$$

where the weights α_i's are non-negative and satisfy,