Optimal Control of Switched Linear Systems by Particle Swarm Optimization

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Abstract: This paper deals with Optimal Control problems of Dynamical Switched Linear Systems (OCPDSLS). This special class of hybrid systems combines the standard control, where dynamic sub-systems are described by differential linear equations with a switching law which activates, alternatively, these subsystems. For a pre-specified modal sequence, we demonstrate that switching time instants and continuous input can be solved simultaneously by using an evolutionary computation technique, namely Particle Swarm Optimization (PSO). Some structural and numerical difficulties encountered in solving such problems by conventional approaches are discussed. The mapping between PSO and OCPDSLS is established via a generic procedure to synthesize optimal control laws. An illustrative numerical non-convex example is solved showing the effectiveness of the proposed technique in overcoming difficulties of this problem known to be NP-hard, non-linear and constrained one.

Keywords: Switched Linear Systems, Switching sequence, Optimal Control, Optimization, Conventional Approaches, Particle Swarm Optimization (PSO).

1. INTRODUCTION

Switched Linear Systems as a particular class of hybrid systems have been under intensive investigations during the last few years. Due to their many potential applications, they have attracted researchers in the fields of modelization [Branicky et al, 98], identification [Juloski et al, 05], observation [Saadaoui et al, 06] and optimal control [Xu, 02 and 04], [Riedinger et al, 05], etc ... This special class of systems involves interaction between continuous and discrete dynamics and has applications in various fields such as mechanical systems, automotive industry, chemical engineering and switching power converters [Benmansour et al, 07].

Recently, Optimal Control Problem of Dynamical Switched Linear Systems (OCPDSLS) has become a challenging research topic because of the large classes of phenomena they describe and the integration of computers in industrial processes control. These problems do not need only the solution of optimal continuous inputs, as in a conventional optimal control problem, but also the solution of optimal modal sequence and its corresponding set of switching instants. The basic difficulty of this kind of problems is the complexity growing exponentially on the number of the handled data, when they are solved by conventional approaches [Xu, 02 and 04].

In [Xu, 02], OCPDSLS has been solved via non-linear optimization based on direct differentiation of the objective function. The main difficulty encountered by this approach is the non-availability of information about derivatives of cost function with respect to switching instants. The General Switched Linear Quadratic (GSLQ) problem discussed in [Xu, 04] is transformed into an equivalent parameter selection problem parameterized by switching instants. For the two-stage proposed algorithm, it is obvious that the optimization of the switching instants may be usually non-convex and the Newton iteration algorithm will easily fall in local minima. In [Luus, 03], the GSLQ problem is considered via a two-level algorithm. The high level, based on a direct search optimization, is devoted to searching the optimal switching instants. The low level based on an Iterative Dynamic Programming (IDP) procedure is used to compute optimal continuous input. The main difficulty encountered by the proposed approach is the combinatorial explosion due to the time and state spaces discretization.

In this paper, we aim to apply a new evolutionary computation technique, namely PSO, to solve the optimal control of linear switched systems problem. Given a pre-specified active sub-systems, one needs to seek both the optimal switching locations, over a finite time horizon, and the corresponding continuous input. For a given switching law, the continuous input has been demonstrated to be a state feedback law. Through time and state spaces discretization, we convert the original problem to a parameter selection one.

The switching instants and the gains of the continuous input law are coded as positions of the particles in the PSO paradigm. These particles freely fly through a multi-dimensional search space when looking for the optimal solution. A suitable mapping between PSO and OCPDSLS allows handling the continuous and the discrete variables together. Without requiring any regularity of the studied problem, the PSO algorithm provides global sub-optimal solutions and allows overcoming the difficulties discussed above.

The paper is organized as follows. In section 2, we propose a unified modelling framework of switched linear continuous-time systems and the general optimal control problem for this class of systems is formulated. To be well-handled by the proposed optimization technique, we propose, an equivalent discrete-time formulation via a discretization procedure of the sub-systems, the switching law and the cost function to be minimized. Section 3 is devoted to the presentation of the PSO technique. In section 4, a generic procedure to compute sub-optimal solutions for the optimal control problem is addressed. We focus on the adaptations of the PSO-based algorithm used in the aim of overcoming the difficulties discussed in section 1. Section 5 is devoted to a literature benchmark, used in optimal control illustrations, to demonstrate the effectiveness of the proposed procedure. Finally, we draw some conclusions in section 6.

2. MODELLING AND OPTIMAL CONTROL OF A SWITCHED LINEAR SYSTEM

3.1 A continuous-time formulation

In this paper, we consider linear switched systems consisting of subsystems, in the state-space continuous-time formalism:

$$\begin{aligned} \dot{x}(t) &= A_q x(t) + B_q u(t) \\ q &\in Q = \{1,\dots,M\} \end{aligned} \tag{1}$$

A_q and B_q are matrices with suitable dimensions and Q indicates that the system will be in M configurations.

$x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ are respectively the state vector and the continuous control input. For such a system, given the modal sequence, $\{q_j \in Q, j = 1,\dots,K\}$ where K is finite, we define the sequence of active subsystems in a time interval $[t_0, t_f]$ by σ, which, from a control point of view, is viewed as a discrete input :

$$\sigma = \{(q_1,t_1),\dots,(q_K,t_K)\} \tag{2}$$

Assumption1: The global system remains in a configuration q_j during, at least, a time $\tau_{\min}$, known as the dwell-time.

The switching instants are constrained by:

$$t_0 < t_1 < \dots < t_K < t_f = t_{K+1} \tag{3}$$

Note that the system switches from subsystem q_j to subsystem $q_{(j+1)}$ at the switching instant t_{q_j}. During the interval $\left[t_{q_{j-1}}, t_{q_j}\right[$ the configuration q_j is active. A collection of states, x, continuous controls, u, and a sequence σ in $[t_0, t_f]$ is known as an execution of the switched system.

Assumption 2: We consider only non-Zeno executions.

Assumption 3: The switched system is supposed to be without jumps.

Problem P1: We consider a switched system as described by (1). Given an interval of time $[t_0, t_f]$, an initial state, x_0 and a sequence of active subsystems $\{q_1,\dots,q_K\}$, an algorithm to solve an optimal control problem of this system has to find the optimal control continuous input, u^* and the optimal switching instants $(t^*_1,\dots,t^*_K)$ such that the following cost-function

$$J(\sigma,u) = \psi(x(t_f)) + \sum_{j=1}^{j=K} \int_{t_{j-1}}^{t_j} L_{q_j}(x,u)dt \tag{4}$$

is minimized. ψ represents the terminal part of this cost function.

If the cost function is quadratic, using a method similar to conventional quadratic optimal control problem, the solution of the Hamilton-Jacobi-Bellmann equations [Xu, 04], provides the continuous input as:

$$u(x,t,q_j) = -G_{q_j}(t)x(t) + E_{q_j}(t), \text{ j=1,...,K} \tag{5}$$

The continuous input is then obtained through its state-feedback control law (5). The problem of computing the optimal continuous input is converted to a parameter selection one involving the gains of this law.

Assumption 4: we assume that the gains vectors $G_{q_j}(t)$ and $E_{q_j}(t)$, j=1,...,K, are constant for each active sub-system q_j.

This assumption 4 is made in order to reduce the numerical complexity of the problem.

3.1 An equivalent discrete-time formulation

The interval of time $[t_0, t_f]$ is divided into N sampling instants such that:

$$N = \frac{t_f - t_0}{T} \tag{6}$$

T is the sampling period.

Assumption 5: According to Shannon sampling theorem, the sampling period T is chosen such that its value is less than