Switching Controller Design for Nonholonomic Four-Wheeled Vehicles via Finite-Time Stabilization Method

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Abstract: In this paper, we study the stabilization problem of nonholonomic four-wheeled vehicles with state constraints. Different from the existing approach where usually the system is transformed into a chained form or a nonholonomic integrator, we deal with the system directly without any transformation, by using finite-time stabilization method repeatedly in a piecewise manner. The main control strategy is to divide the whole procedure into five stages, and to design a finite-time stabilizing controller in each stage so that the vehicle's steering angle and attitude angle reach desired values. The desired values are computed depending on the vehicle's states so that the vehicle moves between two switching lines (corresponding to the state constraints) and finally reaches the desired terminal point.

Keywords: Nonholonomic systems, four-wheeled vehicles, finite-time stability, finite-time stabilization, switching control.

1. INTRODUCTION

In this paper, we consider stabilization of a class of nonholonomic systems, namely four-wheeled vehicles with state constraints. It is known that generally nonholonomic systems do not satisfy the so-called Brockett's stabilizability condition (Brockett, 1983), and thus they can not be asymptotically stabilized to their equilibrium points by any continuously differentiable, time invariant, state feedback control laws (Brockett, 1983; Bloch *et al.*, 1992). For this reason, there have been a large quantity of works on the stabilization problem of nonholonomic systems in the last two decades, including the efforts of finding continuous, time varying control laws (Pomet, 1992; M'Closkey & Murray, 1993), discontinuous ones (Bloch *et al.*, 1992; de Wit & Sørdalen, 1992; Astolfi, 1996) and middle strategies (discontinuous and time varying) (Sørdalen & Egeland, 1993; M'Closkey & Murray, 1995). Another important approach is the hybrid control method proposed by Hespanha & Morse (1999), where four controller candidates and a switching strategy are proposed for nonholonomic integrators and as a result exponential stability is achieved. However, to utilize the approach in Hespanha & Morse (1999) for the present four-wheeled vehicle, one needs to transform the system into a chained form and then a nonholonomic integrator form, while these forms can not deal with the case of involving state constraints.

For this purpose, we here propose a switching control strategy without making any system transformation. The idea is based on finite-time stabilization method. It is known that finite-time stability is totally different from Lyapunov stability in the sense that it requires a system's state should reach an equlibrium point in finite time. It has been shown in Bhat & Bernstein (2000) that finite-time stabilization method is effective for a wide class of nonholonomic systems. Motivated by the approach and the observation in Bhat & Bernstein (2000), we seek the possibility of dealing with state constraints in nonholonomic four-wheeled vehicle systems by using finite-time stabilization method.

The main design strategy of our switching controllers is to divide the control procedure into five stages, and to design a finite-time stabilizing controller in each stage so that the vehicle's steering angle and attitude angle reach desired values. The desired values are computed in a constructive manner corresponding to the vehicle's states so that the vehicle moves between two switching lines and finally reaches the desired terminal point.

The remainder of this paper is organized as follows. In Section 2, we give some preliminaries mainly concerning finite-time stability and stabilization method. In Section 3, we describe the four-wheeled vehicle system and the control problem on hand. Section 4 is devoted to detailed description of the switching control procedure based on finite-time stabilization method. Then, Section 5 presents a numerical example, and finally Section 6 concludes the paper.

2. PRELIMINARIES

In this section, we briefly summarize the concept and some results of finite-time stability and stabilization (or finite-time stabilizing controller) for systems described by differential equations. More details in this area can be found in Bhat & Bernstein (2000) and Haimo (1986). The descriptions here are also based on these references.

Consider an autonomous system described by

$$\dot{x} = f(x) \tag{1}$$

where $x \in \mathcal{R}^n$, $f : D \to \mathcal{R}^n$ is a local map from an open set $D \subset \mathcal{R}^n$ into $\mathcal{R}^n$. Also assume that $x_e = 0$ is an equilibrium point of the system, i.e., $f(0) = 0$.

Let the solution of (1) with the initial condition $x(0) = x_0$ be $x(t; 0, x_0)$. Then, the zero equilibrium point of (1) (or simply the system (1)) is called *finite-time stable (convergent)* if there is an open neighborhood of $U \subset D$ including the origin and a function $T_x : U \to (0, \infty)$, such that for $\forall x_0 \in U$, the solution $x(t; 0, x_0)$ is well defined on $[0, T_x(x_0))$ but $x(t; 0, x_0) \in U \setminus \{0\}$, and $\lim_{t \to T_x(x_0)} x(t; 0, x_0) = 0$. When the above is true, the system (1) is called *a finite-time differential equation.*

An example of finite-time stable system is

$$\dot{x} = -x^{\alpha} \tag{2}$$

where x is a scalar variable, $\alpha = \frac{a}{b}$ and a, b are positive odd numbers satisfying $a < b$. Then, the solution of (2) is

$$x(t) = \begin{cases} \pm[-(1-\alpha)t + C]^{\frac{1}{1-\alpha}} & (t \le T) \\ 0 & (t \ge T) \end{cases}$$

where $C = [x(0)]^{1-\alpha}$ and $T = \frac{C}{1-\alpha}$. It is obvious that the solution reaches zero in finite-time T depending on the initial value and the solution is well defined in forward time. Fig. 1 depicts the state trajectory in case of $\alpha = \frac{1}{3}$ and $x(0) = 5$.

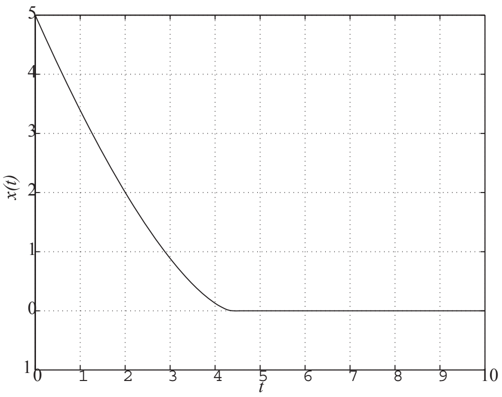

Fig.1. Finite-Time Convergence of $\dot{x} = -x^{\frac{1}{3}}$

Note that for the system (2) the finite time $T = \frac{C}{1-\alpha}$ depends on the initial state $x(0)$ and the parameter α. For example, for any fixed initial state $x(0)$, the finite time with $\alpha = \frac{1}{3}$ is larger than that with $\alpha = \frac{1}{5}$. This observation will be used later to adjust convergence time for four-wheeled vehicles.

The following theorem is used to prove the finite-time stability of differential equations, which can be regarded as a Lyapunov stability theory based condition.

Lemma 1. Suppose that there exists a continuous function $V : D \to \mathcal{R}^+$ such that

- V is positive definite.
- There exist real numbers $c > 0$ and $\alpha \in (0, 1)$ and an open neighborhood $U \subset D$ including the origin such that
$$\dot{V}(x) + c\,(V(x))^{\alpha} \le 0\,, \quad \forall x \in U \setminus \{0\}\,.$$

Then the origin is a finite-time stable equilibrium of (1).

Next, it is natural to define finite-time stabilization and stabilizability of the system

$$\dot{x} = f(x, u) \tag{3}$$

where x is the same as in (1) and $u \in \mathcal{R}^m$ is the control input. If there exists a state feedback $u = \psi(x)$ such that the closed-loop system $\dot{x} = f(x, \psi(x))$ is finite-time stable, then we say the system (3) is *finite-time stabilizable* (via state feedback). In that case, we call the state feedback $u = \psi(x)$ *a finite-time stabilizing controller.*

In the end of this section, we review an important result for finite-time stabilization established in Bhat & Bernstein (1998). The objective is to seek a continuous time-invariant feedback controller $u = \psi(x, y)$ for the double integrator system

$$\dot{x} = y\,, \quad \dot{y} = u \tag{4}$$

such that the closed-loop system is finite-time stable.

Lemma 2. The origin of (4) is globally finite-time stable under the feedback control law

$$u = -\mathrm{sgn}(y)|y|^{\alpha} - \mathrm{sgn}(\phi_{\alpha}(x, y))|\phi_{\alpha}(x, y)|^{\frac{\alpha}{2-\alpha}} \tag{5}$$

with arbitrary $\alpha \in (0, 1)$, where

$$\phi_{\alpha}(x, y) \equiv x + \frac{1}{2-\alpha}\mathrm{sgn}(y)|y|^{2-\alpha}\,.$$

The above result is extended to a rigid body rotating about a fixed axis with unit moment of inertia, described by

$$\ddot{\theta} = u \tag{6}$$

where θ is the angular displacement from some reference and u is the control torque. Eqn. (6) can be rewritten in the form of (4) by substituting $x = \theta$ and $y = \dot{\theta}$. However, for every $(x, y) \in \mathcal{R}^2$, the states $(x \pm 2n\pi, y)$, $n = 0, 1, 2, \cdots$, correspond to the same physical state of the rigid body, and thus we need to require that $(\pm 2n\pi, 0)$, $n = 0, 1, 2, \cdots$, should be finite-time stable. To achieve this goal, the feedback controller (5) should be modified as

$$u = -\mathrm{sgn}(y)|y|^{\alpha} - \mathrm{sgn}(\sin(\phi_{\alpha}(x, y)))|\sin(\phi_{\alpha}(x, y))|^{\frac{\alpha}{2-\alpha}}\,. \tag{7}$$

The controllers (5) and (7) will be repeatedly used with some modification later in Section 4.

3. PROBLEM FORMULATION

The nonholonomic four-wheeled vehicle under consideration is depicted in Fig. 2, where the distance between the front wheels and the rear ones is L, the position of the middle point of the rear wheels is (x, y), the vehicle's attitude angle is θ and the steering angle is ϕ satisfying a reasonable constraint $|\phi| \le \phi_{max}$. Assume that the vehicle's velocity is v, the steering angular velocity is ω. Furthermore, assume that the control inputs are the steering angular acceleration u_1 and the velocity's acceleration