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Disturbance Decoupling with Stability for Linear Impulsive Systems

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Abstract: The paper deals with the problem of decoupling the output of an impulsive linear system from a disturbance input by means of a stabilizing state feedback. Geometric methods are used to characterize the decoupling requirement from a structural point of view and in terms of feedback stabilizability. Assuming that the length of the time intervals between consecutive impulsive variations of the state, or jumps, is lower bounded by a positive constant, necessary and sufficient conditions for the existence of solutions are given and discussed.

Keywords: Hybrid systems, decoupling problems, feedback stabilization

1. INTRODUCTION

Hybrid dynamical systems whose state evolves according to a LTI dynamics, called the flow dynamics, except at isolated points on the time axis, in which it presents jump discontinuities, can arise in modeling phenomena like e.g. collisions in mechanical systems, the action of switches in electrical circuits or component failures in general situations (see Goebel et al. (2012), Liberzon (2003), Haddad et al. (2006)). Regulation problems and noninteracting control problems for systems of that kind, called linear impulsive systems (see e.g. Lakshmikantham et al. (1989), Yang (2001)), have been considered by many authors under various hypotheses, either assuming that jumps are equally spaced in time (i.e. they arise periodically) or not (see Marconi and Teel (2010), Carnevale et al. (2012a), Carnevale et al. (2012b), Carnevale et al. (2013), Carnevale et al. (2014b), Perdon et al. (2015)). In particular, Perdon et al. (2015) considered the problem of decoupling, by state feedback, the output of a linear impulsive system with periodic jumps from an unknown disturbance, while achieving asymptotic stability of the closed-loop dynamics. Here, we focus on the same problem for dynamical systems of a larger class: namely the class of linear impulsive systems with jumps that occur at unpredictable, possibly non-equally spaced times. More precisely, we look for solutions of the problem that are valid for any possible, a-priori unknown, time sequence of jumps, with the only limitation that the time between two consecutive jumps cannot be shorter than a known threshold.

The approach we follow in dealing with this problem employs geometric methods and tools that derive from the classical geometric approach to linear time-invariant systems Basile and Marro (1992); Wonham (1985). Lately, these methods have been extended to the framework of hybrid systems. In particular, they have been applied to switched linear systems in order to solve disturbance decoupling problems Otsuka (2010); Yurtseven et al. (2012); Zattoni et al. (2014a, 2016a), model matching problems Conte et al. (2014); Zattoni et al. (2014b); Perdon et al. (2016), and output regulation problems Zattoni et al. (2013a,b). Moreover, these techniques have been exploited to solve, more generally, noninteracting control problems for jumping hybrid systems or linear impulsive system in Medina and Lawrence (2006); Medina (2007); Carnevale et al. (2014b,a, 2015); Zattoni et al. (2015); Perdon et al. (2015).

Differently from Perdon et al. (2015), where only a sufficient condition for the solution of the problem was found, the more restrictive requirement that decoupling and stability hold for all possible time sequences of jumps gives us the possibility to state, in Theorem 1, necessary and sufficient conditions for the existence of solutions. Under a mild assumption, which is akin to, but weaker than left invertibility of the flow dynamics, different necessary and sufficient conditions for solvability, that are easier to check than those of Theorem 1, are given in Theorem 2. A stronger sufficient condition, that can be checked by a finite procedure is also given in Theorem 3.

The paper is organized as follows. In Section 2, we introduce the class of systems we consider and we state the control problem we study. In order to deal with the requirement of asymptotic stability of the closed-loop dynamics for any possible sequence of jumps, we give a result about feedback stabilization of the considered linear impulsive systems under the hypothesis that the flow dynamics is reachable. In Section 3, we describe the geometric tools we are going to use and their relevant properties. In Section 4, we state necessary and sufficient conditions for the solution of the problem and we discuss how to test them and to construct practically a solution. Section 5 contains an

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example. Conclusions and indications for future work are given in Section 6.

Notations. The symbols $\mathbb{N}$, $\mathbb{R}$, $\mathbb{R}^+$ are used for the sets of natural numbers (including 0), real numbers, positive real numbers, respectively. Matrices and linear maps between vector spaces are denoted by slanted capital letters like A. Sets, vector spaces and subspaces are denoted by calligraphic capital letters like $\mathcal{X}$. The quotient space of a vector space $\mathcal{X}$ over a subspace $\mathcal{V} \subseteq \mathcal{X}$ is denoted by $\mathcal{X}/\mathcal{V}$. The restriction of a linear map A to an A-invariant subspace $\mathcal{V}$ is denoted by $A|_{\mathcal{V}}$. The vector spaces image and kernel of a linear map A are denoted by $Im\ A$ and $Ker\ A$, respectively. The symbols A^{-1} is used to denote the inverse of the matrix A. For a vector $v \in \mathbb{R}^n$, $\|v\|_E$ denotes the Euclidean norm and, for a matrix A, $\|A\|$ denotes the matrix norm defined by $\|A\| = sup_{x\neq 0} \frac{\|Ax\|_E}{\|x\|_E}$.

2. THE DISTURBANCE DECOUPLING PROBLEM WITH STABILITY FOR IMPULSIVE SYSTEMS

Let us denote by $\mathcal{S}$ the set of all maps $\sigma : \mathbb{N} \to \mathbb{R}^+$ which, letting $\tau_\sigma = inf\{\sigma(0), \sigma(i+1) - \sigma(i); i \in \mathbb{N}, \sigma(i+1) \neq \sigma(i)\}$, satisfy the following conditions

$$\tau_\sigma > 0. \tag{1}$$

Condition (1) implies that $\sigma(i+1)$ is greater than or equal to $\sigma(i)$ for all $i \in \mathbb{N}$ and also that the set of points in the image of σ, i.e. the set $Im(\sigma) = \{t \in \mathbb{R}^+, t = \sigma(i)$ for some $i \in \mathbb{N}\}$, is a discrete, finite or countably infinite subset of $\mathbb{R}^+$, whose subsets (including $Im\,\sigma$ itself) have no accumulation points. We say that τ_σ, which describes the minimum interval between two points in $Im\ \sigma$, is the *dwell time* of σ. In the following, given $\tau \in \mathbb{R}^+$, we will denote by $\mathcal{S}_\tau$ the subset of $\mathcal{S}$ defined by $\mathcal{S}_\tau = \{\sigma \in \mathcal{S}, \tau_\sigma \geq \tau\}$.
The dynamical systems we consider are linear hybrid systems which present jumps in the state evolution, as described by the following equations:

$$\Sigma_\sigma \equiv \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \text{ for } t \neq \sigma(i),\ i \in \mathbb{N} \\ x(\sigma(i)) = Jx^-(\sigma(i)) \text{ for } i \in \mathbb{N} \\ y(t) = Cx(t) \end{cases} \tag{2}$$

where $t \in \mathbb{R}$ is the time variable; $x \in \mathcal{X} = \mathbb{R}^n$, $u \in \mathcal{U} = \mathbb{R}^m$ and $y \in \mathcal{Y} = \mathbb{R}^p$ denote, respectively, the state, the input and the output variables; A, B, J, C are real matrices of suitable dimensions and $x^-(\sigma(i))$ denotes the limit of $x(t)$ for t which goes to $\sigma(i)$ from the left, that is $x^-(\sigma(i)) = lim_{t\to\sigma(i)^-} x(t)$; σ belongs to $\mathcal{S}$. In other words, the state $x(t)$ of Σ_σ, starting from an initial condition $x(0) = x_0$ at time $t = 0$, evolves continuously on the time interval $[0, \sigma(0))$ according to the dynamics given by the first block of equations in (2). Then, at time $t = \sigma(0)$, instead of taking the value $x^-(\sigma(0))$, the state jumps to $Jx^-(\sigma(0))$, as stated in the second block of equations in (2). The same behavior repeats on each one of the subsequent time intervals $[\sigma(i), \sigma(i+1))$, with initial condition $x(\sigma(i))$. We say that the equations in the first block in (2) represent the *flow dynamics* of Σ_σ, while the equations in the second block represent the *jump behavior* of Σ. Jumps occur at all points $\sigma(i) \in Im\ \sigma$ (it is understood that a single jump occurs at $\sigma(i)$, also in case $\sigma(i) = \sigma(j)$) and the dwell time τ_σ represents the minimum interval between consecutive, distinct jump times in Σ_σ. The jump behavior depends on J and on the choice of the map σ and, in particular, it is not necessarily periodic. Note that we do not assume that the interval between consecutive jumps has an upper bound. In case $Im\ \sigma$ is a finite set, no jumps occur for $t > max_{i\in\mathbb{N}}\sigma(i)$ and, on the interval $(max_{i\in\mathbb{N}}\sigma(i), +\infty)$, the dynamics of Σ_σ reduces to the flow dynamics.

Dynamical systems of the above kind are known as linear impulsive systems Lakshmikantham et al. (1989), Yang (2001). Recent results, ensuing from the geometric approach, on their structural and stability properties, under restrictions on the dwell time, are found in Medina and Lawrence (2009), Medina (2007). For impulsive control systems, that is systems of the above kind in which the input acts also on the jump behavior, regulation problems have been considered from a geometric point of view in Carnevale et al. (2014b).

Let us consider a linear impulsive system of the form (2) with an additional unknown disturbance input $d(t)$, that is the system $\Sigma_{\sigma d}$ given by the following equations

$$\Sigma_{\sigma d} \equiv \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Dd(t) \\ \qquad \text{for } t \neq \sigma(i),\ i \in \mathbb{N} \\ x(\sigma(i)) = Jx^-(\sigma(i)) \text{ for } i \in \mathbb{N} \\ y(t) = Cx(t) \end{cases} \tag{3}$$

where $d \in \mathcal{D} = \mathbb{R}^h$, and D is a real matrix of suitable dimensions. Then, the control problem we want to study is described as follows.

Problem 1. **Disturbance Decoupling with Global Asymptotic Stability.** Given a system $\Sigma_{\sigma d}$ of the form (3) and $\tau \in \mathbb{R}^+$, the Disturbance Decoupling Problem with Global Asymptotic Stability (DDPS) for $\Sigma_{\sigma d}$ and τ consists in finding a state feedback $F : \mathcal{X} \to \mathcal{U}$ such that the compensated impulsive system $\Sigma^F_{\sigma d}$ given by

$$\Sigma^F_{\sigma d} \equiv \begin{cases} \dot{x}(t) = (A + BF)x(t) + Dd(t) \\ \qquad \text{for } t \neq \sigma(i),\ i \in \mathbb{N} \\ x(\sigma(i)) = Jx^-(\sigma(i)) \text{ for } i \in \mathbb{N} \\ y(t) = Cx(t) \end{cases}$$

satisfies the following requirements for any $\sigma \in \mathcal{S}_\tau$:

- $\mathcal{R}\,1$. the output $y(t)$ is decoupled from the disturbance input $d(t)$;
- $\mathcal{R}\,2$. the impulsive system $\Sigma^F_{\sigma d}$ is globally asymptotically stable.

Requirement $\mathcal{R}\,1$ can be expressed by saying that $x(0) = 0$ implies $y(t) = 0$ for all $t \in \mathbb{R}^+$ and any disturbance $d(t)$. It has to be remarked that in solving the problem we will be able to characterize completely the set of states for which $y(t) = 0$ for all $t \in \mathbb{R}^+$ and any disturbance $d(t)$. Coming to the requirement of global asymptotic stability for any choice of $\sigma \in \mathcal{S}_\tau$, note that, since it must be satisfied, in particular, for $\bar{\sigma}$ defined by $\bar{\sigma}(i) = \tau$ for all $i \in \mathbb{N}$ (i.e in the case in which the system behavior presents a single jump in $t = \tau$), $\mathcal{R}\,2$ implies that all free motions of the compensated flow dynamics that are initialized in ImJ go asymptotically to 0. Hence, in order to satisfy $\mathcal{R}\,2$, (global) asymptotic stability of the compensated flow dynamics restricted to the smallest invariant subspace $\mathcal{W}$ such that of $Im\ J \subseteq \mathcal{W} \subseteq \mathcal{X}$ is necessary.

The disturbance decoupling problem has been considered in Perdon et al. (2015) for the class of linear impulsive systems of the form (2) in which jumps are equally spaced