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Parametric Identification of Hybrid Linear-Time-Periodic Systems

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Abstract:
In this paper, we present a state-space system identification technique for a class of hybrid LTP systems, formulated in the frequency domain based on input–output data. Other than a few notable exceptions, the majority of studies in the state-space system identification literature (e.g. subspace methods) focus only on LTI systems. Our goal in this study is to develop a technique for estimating time-periodic system and input matrices for a hybrid LTP system, assuming that full state measurements are available. To this end, we formulate our problem in a linear regression framework using Fourier transformations, and estimate Fourier series coefficients of the time-periodic system and input matrices using a least-squares solution. We illustrate the estimation accuracy of our method for LTP system dynamics using a hybrid damped Mathieu function as an example.

Keywords: Linear time periodic systems, state-space identification, hybrid systems.

1. INTRODUCTION

Our main focus in this paper is the identification of nonlinear, hybrid dynamical systems that operate near their periodic solutions. A wide variety of dynamical phenomena in biology and engineering include *oscillatory* and *hybrid* characteristics (Buehler et al., 1994; Chevallereau et al., 2009; Hurmuzlu and Basdogan, 1994). Thus, such dynamical behaviors are commonly modeled as nonlinear hybrid dynamical systems that operate near some isolated periodic orbits (a.k.a. limit-cycle). Though, there are remarkably fewer studies focusing on the problem of system identification for hybrid dynamical systems operating around limit-cycles than system identification studies on-dynamical systems that operate near their point equilibria (e.g. LTI systems).

In the broadest sense, a hybrid dynamical system is one that both flows smoothly (defined by a set of differential equations) and jumps discretely (defined by a set of transition maps) (Guckenheimer and Holmes, 1991). These discrete jumps are often accompanied by a switch between different smooth flows, punctuating system trajectories with discontinuous jumps, sometimes even changing the dimension of the underlying state space (Burden et al., 2015). Despite the generality of this definition, we limit our scope to hybrid systems for which state trajectories are continuous, but possibly non-differentiable. In other words, we exclude systems that undergo discrete jumps in states as well as changes in the state dimensions.

Under certain assumptions, the linearization of smooth nonlinear systems around their periodic solutions (orbit), yields linear time-periodic (LTP) systems (Guckenheimer and Holmes, 1991), whereas the linearization of the class of nonlinear hybrid systems we consider around their periodic orbits yields hybrid LTP systems (DaCunha and Davis, 2011). Since we exclude hybrid transitions with discrete jumps in system state and dimension, the class of induced hybrid LTP systems that we study exhibit continuous but only piece-wise differentiable vector fields (Uyanik et al., 2015a, 2016). In Section 2.2, we formally define the general form of LTP systems that we focus on. Our main contribution in this paper is a parametric system identification method for hybrid LTP systems that we consider, using frequency domain representations of input-output data.

Unlike the literature on LTP and/or Hybrid system identification, the identification of LTI systems is a relatively mature field. There is a wide range of techniques for the identification of LTI systems, appropriate for widely differing needs of engineers and scientists (Ljung, 1998).

There are a number of methods that extend the LTI identification techniques to the identification of LTP systems. For example Shi et al. (2007) utilizes the subspace system identification method (Van Overschee and De Moor, 1996) to estimate physical parameters of smooth linear time-varying systems, whereas Verhaegen and Yu (1995) developed a different subspace system identification technique for discrete time periodically time-varying systems.

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In the context of piece-wise smooth system identification, Verdult and Verhaegen (2004) introduced a formulation to estimate state space models for piecewise LTI systems, which may be considered as a special case of our formulation in Section 2.2, when the switching time between the subsystems is known. Similarly, Buchan et al. (2013) utilizes a data-driven input–output system identification method to estimate piecewise affine models for approximating dynamics of a hexapedal robot. However, none of these methods completely cover our class of LTP systems and they all perform identification based on time domain input-output data.

In our formulation, we assume that switching times between different continuous LTP vector fields are known. This information is used to separately identify individual contributions from each LTP subsystem to the overall periodic system. In our approach, we obtain Fourier series coefficients for the state and input matrices and then formulate the problem in a linear regression framework. After estimating system matrices using a least squares solution, we use Fourier synthesis to construct time-periodic system and input matrices.

This paper is organized as follows. Section 2 details the formulation of the problem as well as underlying models and assumptions. Section 3 describes the theory behind the estimation of time-periodic system matrices. Section 4 illustrates a case study on a simple example, time-switched damped Mathie function with associated estimation results.

2. PROBLEM FORMULATION

2.1 Linear Time-Periodic Systems

In this paper, we focus on linear time-periodic systems, whose state evaluation equation can be written as

$$\dot{\mathbf{x}}(t) = A(t)\mathbf{x}(t) + B(t)\mathbf{u}(t) \ , \tag{1}$$

where both system matrices are periodic with a fixed, known period $T > 0$ such that $A(t) = A(t + nT)$ and $B(t) = B(t+nT)$, $\forall n \in \mathbb{Z}$. In this study, we further assume that we can measure all system states.

In an LTI system, a sinusoidal input signal with a frequency of ω, at steady state produces output only at the same frequency, possibly at a different phase and magnitude. This is the well-known frequency separation principle of LTI systems, allowing the use of transfer functions to characterize input–output relations for such systems in the frequency domain.

On the other hand, for an LTP system, a sinusoidal input at a specific frequency ω, produces not only an output at the same frequency, ω, but also components at frequencies that are the sum of ω and the harmonics of the pumping frequency $\omega_p = 2\pi/T$ of the system (i.e. at $\omega + k\omega_p, k \in \mathbb{Z}$), all with possibly different magnitudes and phases in steady state. Based on this property, the concept of Harmonic Transfer Functions (HTFs) were developed by Wereley (1991), where distinct transfer functions capture each of these harmonic responses.

Existing literature on frequency domain system identification of LTP systems concentrates mainly on non-parametric estimation of the harmonic transfer functions (HTFs) (Hwang, 1997; Siddiqi, 2001; Louarroudi et al., 2012; Uyanik et al., 2016), Even though a number of previous studies perform parametric identification by fitting parameterized transfer function models to the non-parametrically identified HTFs (Ankarali and Cowan, 2014; Uyanik et al., 2015a,b), the present study focuses on a direct state-space parametric identification method for the hybrid LTP system without dealing with computational details of HTFs.

In this formulation, the steady-state response of the system can be represented as

$$X(j\omega) = \sum_{n=-\infty}^{\infty} H_n(j\omega - jn\omega_p)U(j\omega - jn\omega_p) \ , \tag{2}$$

where $H_n(s)$ can be theoretically derived for certain special cases when the state space representation of the system is available, such as for systems with finite harmonic expansions or constant system matrices (Wereley, 1991; Möllerstedt, 2000).

2.2 Modeling and Assumptions

In this paper, we will work with systems in the form of (1), assumed to be driven by an observable input, $u(t)$, with measurements provided for all of its states. Moreover, we also require the following assumptions to hold.

Assumption 1. Our models of interest consist of M alternating "unknown" LTP sub-dynamics, $A^1(t)$, $A^2(t)$, $\cdots$, $A^M(t)$, whose activations are triggered by M complementary "known" rectangular switching functions, $s^1(t)$, $s^2(t)$, $\cdots$, $s^M(t)$, during each cycle of the system. Both $A^i(t)$ and $s^i(t)$ are T-periodic functions, with the switching functions taking the form

$$s^i(t) = \begin{cases} 1, & \text{if} \quad t_i + nT \leq t < t_{i+1} + nT, \quad \forall n \in \mathbb{Z} \\ 0, & \text{otherwise,} \end{cases} \tag{3}$$

where t_i's denote the known switching times and satisfy the conditions $t_1 = 0$, $t_{M+1} = T$, and $t_i < t_{i+1}, \forall i \in 1, \cdots, M$. The state and input matrices of (1) can hence be written as

$$A(t) = \sum_{i=1}^{M} A^i(t)s^i(t), \quad B(t) = \sum_{i=1}^{M} B^i(t)s^i(t). \tag{4}$$

□

Assumption 2. We assume that the system period T as well as the transition times between different sub-system dynamics can be measured and are known. This information is sufficient to construct the switching functions, $s^1(t)$, $s^2(t)$, $\cdots$, $s^M(t)$, that trigger the activation of alternating sub-systems. □

Based on the LTP framework and our assumptions listed above, the problem we are interested can be defined as:
Given

- a number of single-sine (or sums-of-sines) input measurements applied at different frequencies, $u(t)$,
- corresponding state measurements, $x(t)$,
- the system period, T, and the switching times between successive subsystems, $s^1(t)$, $s^2(t)$, $\cdots$, $s^M(t)$,

Estimate piecewise smooth, linear time-periodic state and input matrices, $A^1(t)$, $A^2(t)$, $\cdots$, $A^M(t)$ and $B^1(t)$, $B^2(t)$, $\cdots$, $B^M(t)$.