Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-9 (2016) 039–044

Internal model-based tracking of a servo gantry system with frequency-varying references

Zhen Zhang [*,**,***] **Han Chen** [****] **Peng Yan** [****,†]

[*] *State Key Laboratory of Tribology & Institute of Manufacturing Engineering, Department of Mechanical Engineering, Tsinghua University, Beijing 100084, China (e-mail: zzhang@ tsinghua.edu.cn).*
[**] *Beijing Key Lab of Precision/Ultra-precision Manufacturing Equipments and Control, Tsinghua University, Beijing 100084, China*
[***] *State Key Laboratory of Fluid Power Transmission and Control, Zhejiang University, Hangzhou 310027, China*
[****] *School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China, (e-mail: pengyan2007@gmail.com)*
[†] *School of Mechanical Engineering, Shandong University, Jinan, Shangdong 250061, China*

Abstract: This paper presents an internal model-based tracking control for a class of nonlinear systems driven by frequency-varying references. It is shown that if the error-zeroing input is polynomial of exosystem state, a linear time-varying internal model can be utilized to generate higher order modes of the exosystem. The proposed control design is applied to a high precision servo gantry system to track frequency-varying references. A numerous experiments are conducted to demonstrate the significant improvement of the tracking performance with higher order internal model.

Keywords: Internal model principle, mechatronics, linear parameter-varying systems.

1. INTRODUCTION

High precision tracking is one of key enabling techniques for a wide applications in precision mechatronics. Significant efforts have been devoted to various aspects of tracking control theory in the past several decades (see, for example Francis (1977); Tsao and Tomizuka (1994); Zhang and Serrani (2006); Zhang and Sun (2010); X. Song and Sun (2012)). As one of the most effective approaches, the internal model principle-based control has emerged as a powerful technique for tracking and/or rejecting exogenous signals generated by autonomous systems.

Although the internal model-based control theory for LTI systems has been well established Francis (1977), the results for LTV (Linear Time-Varying) systems remain open, due to the fundamental challenges of constructing a time-varying internal model to render the error-zeroing subspace invariant, and a robust time-varying stabilizer for the augmented time-varying system.

Note that a systematic design approach for the construction of time-varying internal model has been developed in Sun et al. (2009); Zhang and Sun (2010) in both input/output and state-space representations. Due to the nonlinearities associated with the plants in actual applications, it is desired to extend the results for LTV systems to nonlinear systems, that is consider some class of nonlinear systems driven by exogenous signals generated by time-varying autonomous systems. Very recently some attempts have been made for extending the results to nonlinear valve actuation systems for automotive industry Yong et al. (2014), where a linear time-varying internal model was utilized for plant model satisfying Winer model. Notice that it is not yet clear why a linear internal model is effective in such a case.

Motivated by this, we, in the this paper, consider the similar problem but by means of different analysis. Specifically instead of only considering I/O of the system, we investigate the control input rendering the error-zeroing manifold invariant. For the class of exosystem under consideration and polynomial of exogenous state in the desired control, we could utilize higher order linear time-varying internal model to generate higher modes of the reference. Then the recently developed linear time-varying internal model Zhang et al. (2013, 2014) can be applied for the calculation of the time-varying internal model.

The rest of the paper is organized as follows: in Section 2 the tracking control problem for letting the plant with nonlinearities track frequency-varying references generated by time-varying exosystem is formulated. A higher order linear time-varying internal model is constructed to compensate the nonlinear deformation of the frequency modes of the exosystem, and the resulting robust control design is presented in Section 3. The proposed tracking control design is applied to a high precision servo gentry system in Section 4, with system identification 4.1 and

10.1016/j.ifacol.2016.07.486

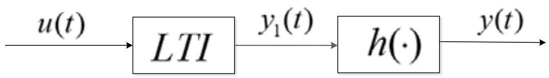

Fig. 1. Wiener model.

experimental results given in Section 4.2 followed by conclusion.

2. PROBLEM DESCRIPTION

Consider the SISO (single input single output) nonlinear plant model of the form

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y_1 &= Cx \\ y &= h(y_1)\,, \end{aligned} \tag{1}$$

where the state $x \in \mathbb{R}^n$, the control input $u \in \mathbb{R}$, unmeasurable $y_1 \in \mathbb{R}$, and the output $y \in \mathbb{R}$. The above model can be considered as one satisfying Wiener model (see Rugh (1981) Figure 1). The tracking control problem under consideration is to let the output $y(t)$ track frequency-varying references $r(t)$ of the form

$$r(t) = a\sin(\sigma(t) + \phi)\,,$$

where $r \in \mathbb{R}$, a, $\sigma(t)$, and ϕ are amplitude, frequency and phase of $r(t)$ respectively. Function $\sigma(t)$ is smooth and bounded. The above reference $r(t)$ can be generated by a tim-varying autonomous system in the form

$$\begin{aligned} \dot{w} &= S(t)w \\ r &= Qw\,, \end{aligned} \tag{2}$$

where

$$\begin{aligned} S(t) &= \sigma(t)S\,, \\ Q &= (1\ \ 0)\,, \end{aligned}$$

with S, a harmonic oscillator.

Remark 2.1. The reference $r(t)$ can be extended to a more general case by augmenting S with multiple harmonic oscillators with different frequencies.

The aim of this paper is to design an internal model principle-based controller such that the output $y(t)$ can track the reference $r(t)$ generated by exosystem (2), that is

$$\lim_{t\to\infty} e(t) = \lim_{t\to\infty} y(t) + r(t) = 0\,.$$

The class of nonlinear systems under consideration satisfies the following assumption.

Assumption 2.1. The system (1) has relative degree n, that is

$$L_g h(x) = \cdots = L_g L_f^{n-2} h(x) = 0$$

and

$$L_g L_f^{n-1} h(x) \neq 0\,.$$

It is well known that by means of a change of coordinates, system (1) can be transformed into the following normal form

$$\begin{aligned} \dot{z}_i &= z_{i+1}\,, \quad i = 1, \cdots, n-1 \\ \dot{z}_n &= f_n(z) + g_n(z)u \\ y &= z_1 \\ e &= z_1 + Qw\,, \end{aligned} \tag{3}$$

which can be rewritten in the following form

$$\begin{aligned} \dot{z} &= A_0 z + B_0[f_n(z) + g_n(z)u] \\ y &= C_0 z \\ e &= C_0 z + Qw\,. \end{aligned}$$

where

$$A_0 = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}, \quad B_0 = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \quad C_0 = (1\ 0\ 0\ \cdots\ 0)\,.$$

The solvability of the tracking problem lies that the existence $\pi(t,w)$ and $c(t,w)$ such that the following regulator equation holds

$$\begin{aligned} \frac{\mathrm{d}\pi}{\mathrm{d}t} + \frac{\partial\pi}{\partial w}\sigma(t)Sw &= A_0\pi(t,w) + \\ &\quad B_0[f_n(\pi(t,w)) + g_n(\pi(t,w))c(t,w)] \\ 0 &= C_0\pi(t,w) + Qw\,. \end{aligned} \tag{4}$$

In practice, it is reasonable to assume that $c(t,w)$ is polynomial in w. The key is to construct input $u_{\mathrm{im}} = c(t,\pi)$, which can be resorted by finding a time-varying system, into which the following autonomous system (5) is immersed.

$$\begin{aligned} \dot{w} &= \sigma(t)Sw \\ v &= c(t,w)\,. \end{aligned} \tag{5}$$

In what follows, we consider the case that function $c(t,w)$ is polynomial in w.

Remark 2.2. Since function $c(t,w)$ is polynomial in w, an internal model should be able to generate

$$\sin(\varphi(t) + \phi_1)\,, \quad \sin(2\varphi(t) + \phi_2)\,, \quad \cdots\,, \tag{6}$$

with $\varphi(t) = \int_{t_0}^{t} \sigma(\tau)\,\mathrm{d}\tau$, $\sigma(t_0) = 0$.

This is similar to the time-invariant polynomial case Khalil (1994); Huang (1995), but one needs to find a time-varying immersion. In the next section, we show how to generate the above modes by resorting linear time-varying internal model.

3. CONTROLLER DESIGN

When the plant model (1) is linear (that is $y = y_1$), the condition (5) reads as

$$\begin{aligned} \dot{w} &= \sigma(t)Sw \\ v &= c(t)w\,. \end{aligned} \tag{7}$$

We have shown in Zhang and Sun (2010); Sun et al. (2009)) that a linear time-varying internal model can be constructed, that is we have provided a system immersion for system (7). Note that

$$\begin{aligned} \dot{w} &= \sigma(t)S_{\mathrm{a}}w \\ r_m &= Qw\,. \end{aligned} \tag{8}$$

where $S_{\mathrm{a}} = \mathrm{diag}(S_1, \cdots, S_m)$, and $Q_{\mathrm{a}} = (Q\,, \cdots, Q)$. is capable to generate multiple modes in (6).

With the above result in mind, we tackle the polynomial nonlinear case (5) by utilizing augmented linear time-varying internal models based on system (8).