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Disturbance Observer Based Optimal Controller Design for Active Suspension Systems

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Abstract: This paper is concerned with the design of disturbance observer based optimal controller for an active control problem of a vehicle suspension system with saturated actuators against magnitude bounded disturbances. A quarter car model presented to analyze performance of proposed controller against bump road profile. Proposed control scheme composed of a disturbance observer and a control law with a state feedback and disturbance feedforward. Gain matrices belong to disturbance observer and the control law are simultaneously computed to minimize L_2 gain of the closed loop system from disturbances to performance outputs by Linear Matrix Inequalities (LMIs) constraints. Finally, numerical simulations are carried out to demonstrate performance of proposed controller.

Keywords: Disturbance observer based control, state feedback, disturbance feedforward, L_2 gain control, active suspension systems.

1. INTRODUCTION

Suspension systems are the main critical parts of vehicles. A successfully designed suspension system should be able to provide the safety of the passengers and protect the vehicle from damage caused by road unevenness (Ulsoy *et al*., 2012). In addition, ground induced vibrations can contribute to reduction of driver's capability to control the vehicle and cause comfort problem (Hrovat, 1997). Therefore, a vehicle suspension system has to attenuate vertical vibrations to improve safety and ride comfort performance.

Vehicle suspension has extensively been studied for a long time (Zhao *et al*., 2010). Especially, the active suspension systems have been attracting the attention in recent years and various approaches have been proposed to improve the damping performance (Williams, 1997). Theoretical limitations and achievable dynamic responses of active suspensions are investigated by several researchers (Karnopp, 1986, Smith, 1995). If accurate measurement of road irregularities is obtained online, performance of an active suspension system can be moved beyond its limitations (Deshpande *et al*., 2014). In the literature, various research studies are reported to measure the road unevenness directly and indirectly. Abdel-Hady (1994) proposed a preview control scheme to sense the road unevenness before reaching the vehicle. On the other hand, some researchers investigated indirect measurement methods to eliminate the need of extra sensors (Chen and Huang, 2005, Deshpande *et al*., 2013). Thereafter various disturbance observer based control schemes have been investigated in active suspension systems. Fialho and Balas (2002) have used road roughness adaptation to avoid the trade-off between suspension deflection and ride comfort. Fuzzy logic controller and disturbance observer combination have been applied by Yoshimura and Takagi (2004). In the control scheme designed by Tahboub (2005), frequency content of a disturbance have been estimated by extended state observer and served as switching rule between linear state feedback controllers. Chen and Tomizuka (2013) proposed an internal model controller which uses an inverse model estimated by an adaptive disturbance observer. Deshpande et al. (2014) combined sliding mode control with disturbance observer to estimate effect of nonlinearities and unknown road disturbances.

All papers discussed above add some significant contributions to the literature. Despite the fact that disturbance observer based control strategies have been used to design active suspension systems, simultaneous computation of disturbance observer and controller gains to minimize L_2 gain of the closed-loop system with LMI constraints has not been considered so far. Furthermore, energy boundedness of control signal against magnitude bounded disturbances is also guaranteed.

Notation: A fairly standard notation is used throughout the paper. $\Re$ stands for the set of real numbers, $\Re^{n\times n}$ is the set of $n\times n$ dimensional real matrices. $\Re_+$ denotes the set of positive real numbers. diag() denotes the diagonal matrices. The identity and null matrices are shown as I and 0, respectively. $X>0\,(<0)$ denotes that X is a positive definite (negative definite) matrix. The notation sym(X) stands for summation of X and its transpose X^T, it can be also showed as $\text{sym}(X)=X+X^T$.

2. MATEMATICAL MODELLING OF ACTIVE SUSPENSION SYSTEM

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In this study, a two degree-of-freedom quarter vehicle model, shown in Fig. 1, is considered for disturbance observer based optimal controller design. m_u is the unsprung mass which represents wheel assembly, m_s is the quarter vehicle sprung mass, which represents the vehicle cabin floor. $x_u(t)$ and $x_s(t)$ are the vertical displacements of unsprung mass and sprung masses, respectively and $x_r(t)$ is the road irregularity.

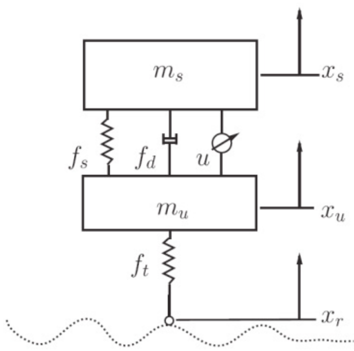

Fig. 1. The quarter vehicle suspension model

$f_s(t)$ is the spring force, $f_d(t)$ is the damping force and $f_t(t)$ is the tyre force and $u(t)$ is the control force generated by the actuator. The spring force is

$$f_s(t) = k_s[x_s(t) - x_u(t)] \tag{1}$$

where the coefficient k_s is the spring coefficient of suspension and the damping force is described as follows,

$$f_d(t) = c_s[\dot{x}_s(t) - \dot{x}_u(t)] \tag{2}$$

where the c_s is damping coefficient of the suspension. The tyre force is calculated as

$$f_t(t) = k_t[x_u(t) - x_r(t)] \tag{3}$$

where k_t is the spring coefficient of tyre. The dynamic vertical motion of equations for the quarter vehicle active suspension are given by

$$m_s\ddot{x}_s(t) + f_s(t) + f_d(t) = u(t) \tag{4}$$

$$m_u\ddot{x}_u(t) - f_s(t) - f_d(t) + f_t(t) = -u(t) \tag{5}$$

In order to design disturbance observer based optimal controller, equations (4) and (5) can be written in state-space form as,

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) \tag{6}$$

where $x(t)$ is the state vector in the form,

$$x(t) = \begin{bmatrix} x_u(t) & x_s(t) & \dot{x}_u(t) & \dot{x}_s(t) \end{bmatrix}^T \tag{7}$$

and the control vector and disturbance vector are assumed to be in the forms, respectively

$$u(t) = \left[u(t)\right] \tag{8}$$

$$w(t) = \left[x_r(t)\right]. \tag{9}$$

Here, $A \in \Re^{n \times x}$ is the system matrix, $B_1 \in \Re^{n \times p}$ is the disturbance input matrix and $B_2 \in \Re^{n \times m}$ is the control input matrix. The state-space matrices of the quarter vehicle suspension model are given as,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\dfrac{k_s + k_t}{m_u} & \dfrac{k_s}{m_u} & -\dfrac{c_s}{m_u} & \dfrac{c_s}{m_u} \\ \dfrac{k_s}{m_s} & -\dfrac{k_s}{m_s} & \dfrac{c_s}{m_s} & -\dfrac{c_s}{m_s} \end{bmatrix} \tag{10}$$

$$B_1 = \begin{bmatrix} 0 & 0 & \dfrac{k_t}{m_u} & 0 \end{bmatrix}^T \tag{11}$$

$$B_2 = \begin{bmatrix} 0 & 0 & -\dfrac{1}{m_u} & \dfrac{1}{m_s} \end{bmatrix}^T \tag{12}$$

In this study the mass, damping and stiffness coefficients are assumed to be as follows (Ando and Suzuki, 1996): m_u= 36 kg, m_s=240 kg, c_s= 980 Ns/m, k_t=160000 N/m and k_s=16000 N/m.

3. DISTURBANCE OBSERVER BASED OPTIMAL CONTROLLER DESIGN BY LMI CONSTRAINTS

In this study, disturbance observer based optimal controller which is designed to mitigate vertical vibrations of vehicle suspension system. The control objectives are to guarantee the closed-loop stability and disturbance attenuation in the sense of L_2 gain. Proposed controller is composed of state feedback and disturbance feedforward integrated with a disturbance observer.

Consider the following LTI system,

$$\dot{x}(t) = Ax(t) + B_1 w(t) + B_2 u(t) \tag{13}$$

$$z_c(t) = C_1 x(t) + D_1 w(t) + D_2 u(t) \tag{14}$$

where $z_c(t)$ is the performance output vector. Then our goal is to design an optimal disturbance observer based optimal controller, expressed as

$$u(t) = Kx(t) + G\hat{w}(t) \tag{15}$$

where $\hat{w}(t)$ is the estimation of unknown disturbance, K and G are state feedback and disturbance feedforward gain matrices, respectively. A disturbance observer must be employed to enable disturbance feedforward. Disturbance observer used in this study is presented by Li *et al*. (2014) as follows:

$$\dot{d}(t) = -LB_1(d(t) + Lx(t)) - L(Ax(t) + B_2 u(t)) \tag{16}$$

$$\hat{w}(t) = d(t) + Lx(t) \tag{17}$$