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Decentralised Stabilisation of Nonlinear Time Delay Interconnected Systems

Xing-Gang Yan * **Sarah K. Spurgeon** **, * **Dongya Zhao** ** **Lei Guo** ***

* *School of Engineering & Digital Arts, University of Kent, Canterbury, Kent CT2 7NT, United Kingdom (e-mails: X.Yan@kent.ac.uk, S.K.Spurgeon@kent.ac.uk)*
** *College of Chemical Engineering, China University of Petroleum, Qingdao 266580, P.R. China (e-mail: dyzhao@upc.edu.cn)*
*** *Key Laboratory of Science and Technology on Aircraft Control for National Defence, Beihang University, Beijing 100191, China (e-mail: lguo@buaa.edu.cn)*

Abstract: A decentralised state feedback control scheme is proposed to stabilise a class of nonlinear interconnected systems asymptotically based on the characteristics of the system structure. Under the condition that all the nominal isolated subsystems have uniform relative degree, the considered class of interconnected systems is transferred to a new interconnected system formed of single input systems, which facilitates the decentralised control design. A new term, weak mismatched uncertainty, is introduced for the first time to recognise a class of mismatched uncertainties in the isolated subsystems. The study shows that the effects of both matched and weak mismatched uncertainties in the isolated subsystems can be rejected completely by appropriate choice of control, and the effects of matched interconnections can be largely reduced if the control gain is sufficiently high.

Keywords: Nonlinear interconnected systems, decentralised control, time delay, relative degree.

1. INTRODUCTION

With advances in technology, systems are frequently networked together to form large scale interconnected systems in order to fulfil complex requirements. Such systems appear in power networks, urban traffic networks, ecological systems and energy systems and the corresponding large scale systems are usually distributed geographically in space. This may cause issues with economic cost and reliability of communication links and thus provides an impetus for considering decentralised control which only uses local information within the design (Bakule, 2008).

Decentralised control strategies have been studied for many years and there are numerous results concerning the development of decentralised schemes for interconnected systems including decentralised output feedback schemes (Yan et al., 1998; Bakule, 2008). Much existing work considers interconnected systems with either linear isolated subsystems or linear interconnections, and/or it is required that the nonlinearity or uncertainties satisfy linear growth conditions (Mahmoud, 2009; Ye et al., 2012; Wang et al., 2015). Time delay is another important factor which brings additional complexity to the study of large scale interconnected systems (Yan et al., 2013). A class of time delay interconnected systems is considered in Mahmoud and Bingulac (1998) where delay does not appear in the interconnections. However, the interconnections between two or more physical systems are often accompanied by phenomena such as material transfer, energy transfer and information transfer, which, from a mathematical point of view, can be represented by delay elements (Michiels and Niculescu, 2007). This has motivated the study of large scale time delay interconnected systems (Bakule, 2008; Hua et al., 2008; Ye et al., 2012).

Due to the richness of nonlinear phenomena, there is no general approach to deal with nonlinear systems as for the linear case. It is necessary to study a class of systems and employ the system structure to complete the design for complex systems. A class of interconnected systems with similar structure is considered in Yan and Zhang (1997) where delay is not involved. A class of nonlinear interconnected systems with triangular structure is considered in Hua et al. (2008), and a large scale system composed of a set of single input single output subsystems with dead zone input is considered in Zhou (2008). In both Hua et al. (2008) and Zhou (2008), the control schemes are based on dynamical feedback which increases the computation greatly. A class of feedforward nonlinear systems are considered in Ye et al. (2012) in which the developed results are only applicable to systems with relatively small delay.

In this paper, a decentralised time delay dependent state feedback controller is synthesised to stabilise a class of large scale time delay interconnected systems with unknown nonlinear interconnections. The interconnections are separated into matched and mismatched parts and

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dealt with separately to reduce the conservatism. Geometric transformations are employed to explore the system structure and transfer the nominal isolated subsystems to a set of single input subsystems to facilitate the design. Discontinuous decentralised controllers are designed using the bounds on the uncertainties relating to each component instead of the bounds on the vector of uncertainties. Using the Lyapunov Razumikhin approach, a set of sufficient conditions are derived so that the corresponding closed-loop systems are uniformly asymptotically stable. The concept of weak mismatched uncertainty is introduced. The study shows that all the matched and weak mismatched uncertainties in the isolated subsystems can be completely rejected by appropriately designed control. Moreover, the effects of the matched interconnections can be largely rejected if the control gain is sufficiently high.

Notation: In this paper, $\mathcal{R}^+$ denotes the nonnegative set of real numbers $\{t \mid t \geq 0\}$. For a square matrix $A \in \mathcal{R}^{n\times n}$, the expression $A > 0$ represents that the matrix A is symmetric positive definite and the symbol $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) represents its maximum (minimum) eigenvalue. Suppose the function $g : \mathcal{R}^n \mapsto \mathcal{R}$ is differentiable, and $f := (f_1(\cdot), f_2(\cdot), \cdot, f_n(\cdot))^T : \mathcal{R}^n \mapsto \mathcal{R}^n$. The notation $L_f g(x)$ denotes the derivative of $g(x)$ along f defined by $L_f g(x) := \sum_{i=1}^{n} \frac{\partial g}{\partial x} f(x)$ and $L_f^k g(x)$ represents a recursion defined by $L_f^k g(x) := \frac{\partial L_f^{k-1} g}{\partial x} f(x)$ with $L_f^0 := g(x)$. Finally, $\|\cdot\|$ denotes the Euclidean norm or its induced norm.

2. PROBLEM FORMULATION

Consider a nonlinear interconnected system described by

$$\dot{x}_i = f_i(x_i) + g_i(x_i)\left(u_i + \phi_i(t, x_i, x_{id_i})\right) + \xi_i(t, x_i, x_{id_i}) + \psi_i(t, x, x_d), \qquad i = 1, 2, \cdots n \tag{1}$$

where $x := \mathrm{col}(x_1, \ldots, x_n) \in \mathcal{X} := \mathcal{X}_1 \times \cdots \times \mathcal{X}_n$, $x_i \in \mathcal{X}_i \subset \mathcal{R}^{n_i}$ is the system state of the i-th subsystem ($\mathcal{X}_i$ is a neighborhood of the origin) and $u_i \in \mathcal{R}^{m_i}$ is the input to the i-th subsystem. $x_d := \mathrm{col}(x_{1d_1}, \ldots, x_{nd_n})$ with $x_{id_i} := x_i(t-d_i)$ denote delayed state vectors where $d_i := d_i(t)$ represent a time varying delay which is assumed to be known and bounded by $\overline{d}_i := \sup_{t\in\mathcal{R}^+}\{d_i(t)\} < +\infty$. The initial conditions relating to the time delays are given by $x_i(t) := \varrho_i(t)$ for $t \in [-\overline{d}_i, 0]$ where $\varrho_i(\cdot)$ are continuous. The function matrices $g_i(x_i) := [g_{i1}(x_i), g_{i2}(x_i), \ldots, g_{im_i}(x_i)] \in \mathcal{R}^{n_i\times m_i}$ describe the input distributions. The terms $\phi_i(\cdot) \in \mathcal{R}^{m_i}$ represent the uncertainty in the input channel of the i-th subsystems, and $\xi_i(\cdot) \in \mathcal{R}^{n_i}$ denote the mismatched uncertainties in the i-th isolated subsystems. The terms $\psi_i(\cdot) \in \mathcal{R}^{n_i}$ are unknown interconnections of the i-th subsystem. All the vector fields $f_i(x_i) \in \mathcal{R}^{n_i}$ and $g_{il}(\cdot) \in \mathcal{R}^{n_i}$ are assumed to be smooth enough for $i = 1, \ldots, n$ and $l = 1, \ldots, m_i$.

Definition 1. Consider system (1). The system

$$\dot{x}_i = f_i(x_i) + g_i(x_i)(u_i + \phi_i(t, x_i, x_{id_i})) + \xi_i(t, x_i, x_{id_i}), \qquad i = 1, \ldots, n \tag{2}$$

is called the i-th isolated subsystem of (1), and the system

$$\dot{x}_i = f_i(x_i) + g_i(x_i)u_i, \qquad i = 1, \ldots, n \tag{3}$$

is called the i-th nominal isolated subsystem of system (1).

Definition 2. System (3) is said to have uniform relative degree $(r_{i1}, r_{i2}, \ldots, r_{im_i})$ in domain $\mathcal{X}_i$ if there exist functions $h_{ij}(x_i)$ defined in $\mathcal{X}_i$ such that for any $x_i \in \mathcal{X}_i$,

i) $L_{g_{ij}} L_{f_i}^{k_i} h_{il}(x_i) = 0$ for all $j, l = 1, 2, \ldots, m_i$ and $i = 1, 2, \ldots, n$

ii) the $m_i \times m_i$ function matrices $\Lambda_i(\cdot)$ defined by

$$\Lambda_i := \begin{bmatrix} L_{g_{i1}} L_{f_i}^{r_{i1}-1} h_{i1}(\cdot) & \cdots & L_{g_{im_i}} L_{f_i}^{r_{i1}-1} h_{i1}(\cdot) \\ L_{g_{i1}} L_{f_i}^{r_{i2}-1} h_{i2}(\cdot) & \cdots & L_{g_{im_i}} L_{f_i}^{r_{i2}-1} h_{i2}(\cdot) \\ \cdots & \cdots & \cdots \\ L_{g_{i1}} L_{f_i}^{r_{im_i}-1} h_{im_i}(\cdot) & \cdots & L_{g_{im_i}} L_{f_i}^{r_{im_i}-1} h_{im_i}(\cdot) \end{bmatrix}$$

are nonsingular for $i = 1, \ldots, n$

Remark 1. The definition of uniform relative degree above is from Isidori (1995). The uniform relative degree implies that, for any point $x_i \in \mathcal{X}_i$, the system has relative degree, and the relative degree is independent of $x_i \in \mathcal{X}_i$. A full discussion about relative degree appears in Isidori (1995).

Assume that the i-th nominal isolated subsystem of the interconnected system (1) has uniform relative degree $(r_{i1}, r_{i2}, \ldots, r_{im_i})$ in the domain $\mathcal{X}_i$ for $i = 1, 2, \ldots, n$. The objective of this paper is to design a decentralised control

$$u_i = u_i(t, x_i, x_{id_i}), \qquad i = 1, 2, \ldots, n \tag{4}$$

such that the corresponding closed-loop systems formed by applying (4) to the interconnected system (1) are uniformly asymptotically stable. This problem is called decentralised state feedback stabilisation. The controllers u_i in (4) depend on the local states x_i and delayed states x_{id_i} but are independent of x_j for $j \neq i$, and are called decentralised time delay dependent controllers.

3. INTERCONNECTED SYSTEM ANALYSIS

Consider the interconnected system (1). The distributions generated by $g_{i1}(\cdot), g_{i2}(\cdot), \ldots, g_{im_i}(\cdot)$ in the domain $\mathcal{X}_i$ are denoted by

$$\mathcal{G}_i(x_i) := \mathrm{span}\{g_{i1}(x_i), g_{i2}(x_i), \ldots, g_{im_i}(x_i)\} \tag{5}$$

for $i = 1, 2, \ldots, n$.

In this section, it is assumed that the nominal isolated subsystem (3) has uniform relative degree

$$(r_{i1}, r_{i2}, \cdots, r_{im_i})$$

and the distribution $\mathcal{G}_i(x_i)$ defined in (5) is involutive in the domain $\mathcal{X}_i$ for $i = 1, 2, \ldots, n$. The objective is to transfer the system (1) to a new interconnected system to facilitate design.

Let $r_i := \sum_{l=1}^{m_i} r_{il}$ for $i = 1, 2, \ldots n$. From the definition of relative degree, r_{il} are nonnegative constants and $r_i \leq n_i$. Then, the differentials $dh_{il}(x_i)$, $dL_{f_i} h_{il}(x_i)$, $\cdots$, $dL_{f_i}^{r_{il}-1} h_{il}(x_i)$ are linearly independent for $l = 1, 2, \ldots, m_i$ and $i = 1, 2, \ldots, n$. Let

$$z_{il} := \begin{bmatrix} h_{il}(x_i) \\ L_{f_i} h_{il}(x_i) \\ \vdots \\ L_{f_i}^{r_{il}-1} h_{il}(x_i) \end{bmatrix}, \quad l = 1, 2, \ldots m_i \tag{6}$$

for $i = 1, 2, \ldots, n$. Since the distributions $\mathcal{G}_i(x_i)$ are involutive for $i = 1, 2, \ldots, n$, there exist $n_i - r_i$ functions