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Robust Fractional Order PI Controller Tuning Based on Bode's Ideal Transfer Function

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Abstract: This paper presents a simple analytical method for tuning the parameters of fractional order PI (FOPI) controllers based on Bode's ideal transfer function. The proposed technique is applicable to stable plants describable by a fractional order counterpart of first order transfer function without time delay. Tuning rules are given in order to improve the robustness of the compensated system in the presence of gain uncertainty in the plant model. Finally, the designed FOPI controller is implemented on a laboratory scale twin rotor helicopter and comparison results are provided to show the effectiveness of the proposed tuning rules.

Keywords: Fractional order PI controller, Bode's ideal transfer function, gain uncertainty, tuning rule, twin rotor helicopter.

1. INTRODUCTION

The existence of uncertainty in the plant model is one of the most important issues which should be considered in the design procedure of a practical control system. Nowadays, various techniques have been proposed for designing robust control systems such as the methods based on quantitative feedback theory, the methods based on H_2 and H_∞ optimization and the μ synthesis methods (Morari and Zafiriou, 1989; Zhou and Doyle, 1998).

In recent years, fractional calculus techniques are well used to design of robust control systems such as handling parametric uncertainties in the gain (Li et al., 2010; Luo and Chen, 2009; Badri and Tavazoei, 2014; Luo et al., 2014, 2010) and in the constant coefficient (Monje et al., 2004; Basiri and Tavazoei, 2015; Feliu-Batlle and Castillo-García, 2014; Jin et al., 2011) of the plant model. Also, Lan and Zhou (2013) present a method to design of non-fragile observer based robust control for fractional order nonlinear uncertain systems. Nevertheless, Padula and Visioli (2016) show that the fractional order PID (FOPID) controllers are more fragile in comparison with the conventional PID controllers and therefore, a special attention should be paid in their tuning by the designer.

In (Li et al., 2010; Luo and Chen, 2009; Badri and Tavazoei, 2014; Luo et al., 2014, 2010), iterative optimization strategies are proposed to design fractional order PD (FOPD), fractional order PI (FOPI), and integral-fractional derivative (IFD) controllers based on the desired phase margin (PM), desired gain crossover frequency, and flatness of the Bode phase plot at the desired gain crossover frequency to achieve robustness against gain uncertainty in the plant model. In (Monje et al., 2008; Saidi et al., 2014; Vinagre et al., 2007; Yeroglu and Tan, 2011), numerical nonlinear optimization methods for tuning the parameters of FOPID controllers are proposed to satisfy five design criteria such as gain crossover frequency, PM, flatness of the phase plot at the desired gain crossover frequency, disturbance rejection, and high frequency noise attenuation. Also, Azarmi et al. (2015a) presents a simple method to design Smith predictor based FOPID controllers which compromises between different control objectives such as PM, gain crossover frequency, disturbance rejection, and high frequency noise reduction. However, PM and gain margin (GM) are two measures of the stability margins that may not lead to acceptable results in the face of plant uncertainties (Bishop, 2002). Yaniv and Nagurka (2004) show that the relaxed sensitivity functions guarantees certain GM and PM of the system, i.e. they are more encompassing measures of robustness than PM and GM.

The aim of this paper is to present a simple analytical method for tuning the parameters of FOPI controllers based on Bode's ideal transfer function. Azarmi et al. (2015) proposes an analytical method for designing the parameters of filtered FOPI controllers to compensate fractional order systems with fractional order $1 \leq \alpha < 2$. In the present work, the proposed tuning rules in (Azarmi et al., 2015) is developed for fractional order systems with fractional order $0 < \alpha \leq 1$. Also by using small gain theorem, it is indicated that the robustness of the compensated system in the low and medium frequency ranges of the control system by applying the designed FOPI controller

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is better than the PI controller in the similar structure. In order to evaluate the proposed method, the designed FOPI controller is implemented on a laboratory scale CE 150 helicopter and the results are compared with the results of applying PI, PID and IFD controllers. Effectiveness and simplicity are key features of the presented method.
The structure of this paper is organized as follows. Some required preliminaries are presented in Section 2. The tuning rules and robust analysis of the designed FOPI controller are described in Section 3. The practical results are given in Section 4. Finally, concluding remarks are provided in Section 5.

2. PRELIMINARIES

The three most popular definitions of the fractional order derivative are Grunwald-Letnikov (GL), Riemann-Liouville (RL), and Caputo definitions. In engineering applications, the Caputo definition is frequently used (Podlubny, 1998). The Caputo definition of the fractional order derivative of order γ is given by

$$ {}_a^C\mathrm{D}_t^{\gamma} x(t) = \frac{1}{\Gamma(n-\gamma)} \int_a^t \frac{x^{(n)}(\tau)}{(t-\tau)^{\gamma-n+1}}\, d\tau \,, \tag{1} $$

where $\gamma\ (n-1<\gamma<n\in N)$ is a positive non-integer number. In (1), $\Gamma(\gamma)$ is the Euler's Gamma function. Also, a and t are respectively the lower and the upper terminals of the integral (Podlubny, 1998).
The Laplace transform of the Caputo fractional order derivative is given by

$$ L\{{}_a^C\mathrm{D}_t^{\gamma} x(t)\} = s^{\gamma}\, L\{x(t)\} - \sum_{j=0}^{n-1} s^{\gamma-1-j}\, x^{(j)}(0)\,, \tag{2} $$

which for zero initial conditions is simplified to

$$ L\{{}_a^C\mathrm{D}_t^{\gamma} x(t)\} = s^{\gamma}\, L\{x(t)\}. \tag{3} $$

FOPID controllers are known as the generalized versions of the traditional PID controllers (Podlubny, 1999). A special case of FOPID controller is FOPI controller which is described as follows.

$$ C(s) = k_p\,(1+\frac{1}{T_i\, s^{\lambda}})\ ,\ 0<\lambda<2\,. \tag{4} $$

In (4), the parameters k_p, T_i, and λ are respectively the proportional gain, the integrator coefficient, and the fractional order of the integrator part of the FOPI controller (Podlubny, 1999).
A fractional order transfer function $H(s)$ is represented as

$$ \begin{aligned} H(s) &= \frac{Q(s^{\beta_k})}{R(s^{\alpha_k})} \\ &= \frac{b_m\, s^{\beta_m} + b_{m-1}\, s^{\beta_{m-1}} + \ldots + b_1\, s^{\beta_1} + b_0\, s^{\beta_0}}{a_n\, s^{\alpha_n} + a_{n-1}\, s^{\alpha_{n-1}} + \ldots + a_1\, s^{\alpha_1} + a_0\, s^{\alpha_0}}, \end{aligned} \tag{5} $$

where $a_k\,(k=0\,,1\,,\ldots\,,n)$ and $b_k\,(k=0\,,1\,,\ldots\,,m)$ are constant coefficients. Also, $\alpha_n>\alpha_{n-1}>\ldots>\alpha_0$ and $\beta_m>\beta_{m-1}>\ldots>\beta_0$ are arbitrary real numbers (Podlubny, 1998). Transfer function (5) would be a commensurate order transfer function when all orders α_k and β_k can be written as integer multiples of a biggest common divisor of $\alpha_k\,(k=0\,,1\,,\ldots\,,n)$ and $\beta_k\,(k=0\,,1\,,\ldots\,,m)$ (Podlubny, 1998). Therefore, the commensurate order transfer function $H(s)$ can be described as follows

$$ H(s) = \frac{\sum_{k=0}^{m} b_k\, s^{k\alpha}}{\sum_{k=0}^{n} a_k\, s^{k\alpha}}, \tag{6} $$

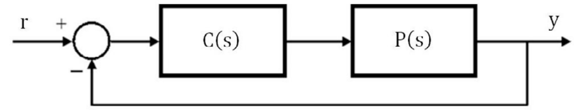

Fig. 1. Closed-loop control system

where α is the biggest common divisor of α_k and β_k (Podlubny, 1998).
Based on the Matignon's stability theorem (Matignon, 1996), the commensurate fractional order transfer function $H(s)$ given in (6) is Bounded Input-Bounded Output (BIBO) stable if

$$ |\angle(w_k)| > \frac{\alpha\pi}{2}\ ,\ k\,=\,1\,,2\,,\ldots\,,n\ ,\ 0<\alpha<2\,. \tag{7} $$

In (7), w_k are the roots of the equation $R(w) = \sum_{k=0}^{n} a_k\, w^k = 0$ built based on the denominator fractional order polynomial of transfer function (6) (Matignon, 1996).

3. FRACTIONAL ORDER PI CONTROLLER DESIGN

As it was mentioned in the litrature review in the present work, the proposed method in (Azarmi et al., 2015) is developed for fractional order systems with fractional order $0<\alpha\le 1$.
Consider the unity negative feedback control scheme shown in Fig. 1 where, $P(s)$ is the process transfer function and $C(s)$ is an FOPI controller which is used to achieve the desired closed-loop transfer function. Consider $P(s)$ as a high order system modelled by a fractional order counterpart of a first order transfer function without time delay. In the rest of this section, the method of designing FOPI controller for such systems is described.

3.1 Design Procedure for the FOPI Controller

According to the structure shown in Fig. 1, the nominal closed-loop transfer function is obtained as

$$ H_r(s) = \frac{y(s)}{r(s)} = \frac{C(s)P(s)}{1+C(s)P(s)}. \tag{8} $$

Consider a high order system modelled by a fractional order transfer function as the following form:

$$ P(s) = \frac{k}{T\, s^{\alpha}+1}\ ,\ 0<\alpha\le 1\ ,\ T>0\,. \tag{9} $$

In (9), k, T, and α are respectively the DC gain, the constant coefficient and the fractional order of the identified model. Different strategies are available in (Tavakoli-Kakhki et al., 2010; Valrio et al., 2008; Victor et al., 2013) for estimating the parameters of the fractional order model (9).
To cancel the stable pole of $P(s)$ (9) with the stable zero of the FOPI controller $C(s)$ (4) with respect to $w=s^{\alpha}$, the fractional order λ and the integrator coefficient T_i of the FOPI controller are respectively considered to be equal to the fractional order α and the constant coefficient T of the identified model (9). Thus, the transfer function of the FOPI controller is chosen as follows.

$$ C(s) = k_p\,(1+\frac{1}{T\, s^{\alpha}}). \tag{10} $$

Based on relations (9) and (10), the open-loop transfer function $L(s)=C(s)P(s)$ is achieved as follows.