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Robust Filtering for Continuous-Time Uncertain Linear Fractional Transformation Systems

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Abstract: This paper is concerned with the problem of robust H_2 and H_∞ filtering for uncertain continuous-time linear systems. The time-invariant uncertain parameters are supposed to belong to a polytope with known vertices and linear fractional transformation (LFT) representation is considered for the uncertainty modeling. Large number of slack variables are used in the proposed method to decouple the Lyapunov variables and the filter parameters. A three-step algorithm is employed to convexify the design problem. The method presented in this paper can be less conservative than the existing methods for the polytopic uncertain systems and its efficiency for filter design is illustrated by means of numerical comparisons with some benchmark examples from the literature.

Keywords: Robust filtering; Polytopic uncertainty; H_2 performance; H_∞ performance; LFT representation

1. INTRODUCTION

The problem of filtering is an important topic in signal processing and control theory. To formulate the filtering problem, the most commonly used performance criteria, namely H_2 and H_∞ norms of the transfer function from the input noise to the output estimation error, are utilized. The problem becomes more complex when uncertainties affect the model and a robust filter should be sought out to keep the performance within acceptable limit Lacerda et al. (2011). The analysis and design of robust filtering and control systems have been investigated extensively over the last two decades whereas the existing methods only provide sufficient conditions Duan et al. (2006). A classical approach is to consider a common Lyapunov function for the entire domain of uncertainty, assuring the so-called quadratic stability of the dynamic system, which causes too much conservatism. To improve the performance, many efforts have been made to reduce the conservativeness of both the analysis and design methods ,see Duan et al. (2006); Geromel (1999); Geromel et al. (2000); Grigoriadis and Watson (1997); Geromel et al. (2002); Xie et al. (2004); Tuan et al. (2001)

Robust filtering problem for linear time-invariant system subject to real parametric uncertainty has attracted remarkable research efforts in recent years. In the context of representing the uncertainty, polytopic uncertainty provides a general framework with great potential applications, that is, the uncertainty domain described by the convex combination of a set of precisely known vertices Sadeghzadeh (2015). Various approaches have been considered to deal with this type of the uncertainty. In Duan et al. (2006), by using an innovative structure for the key slack matrix variables, extra free dimensions in the solution space for the H_2 and H_∞ optimization problems are provided, as an extension of the proposed methods in Xie et al. (2004), Peaucelle et al. (2000).

The problem of robust H_2 and H_∞ filter design for uncertain linear systems with time-invariant parameters belonging to a polytope for continuous and discrete-time systems are investigated in Lacerda et al. (2011). By means of Finslers lemma to introduce slack variables, extra degrees of freedom are provided to reduce the conservativeness.

In Sadeghzadeh (2015) the problem of robust H_2 and H_∞ filter design for time-invariant discrete-time uncertain LFT systems is investigated. Both polytopic and ellipsoidal uncertainties are considered. Using a three-step algorithm, the structure on the auxiliary variables are removed that is indeed promising for achieving less conservative results.

This paper deals with the problem of robust H_2 and H_∞ filter design for continuous time-invariant uncertain LFT systems. The performance criteria are the H_2 and H_∞ norms of the transfer function from the input noise to the output estimation error. A three-step filter design procedure based on LMIs is used. In fact, this paper is the extension of the methods presented in Sadeghzadeh (2015) for continuous-time systems.

The paper is organized as follows. In section 2, the problem formulation and some preliminaries are given. In Section 3,

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the problem of computation of the worst case performance over the uncertainty set for a given filter is investigated. Section 4 is devoted to the robust filter design procedures. Section 5 presents numerical experiments and compares the results of this paper with those of Lacerda et al. (2011); Duan et al. (2006) which explicitly reveals the advantages of the proposed method. Finally, some conclusions are drawn in the last section.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following continuous-time stable system

$$\begin{aligned} \dot{x} &= A_\Delta x + B_\Delta w, \\ y &= C_\Delta x + D_\Delta w, \\ z &= L_\Delta x, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ is the state vector, $y \in \mathbb{R}^{n_y}$ is the measured output, $z \in \mathbb{R}^{n_z}$ is the signal to be estimated, and $w \in \mathbb{R}^{n_w}$ is the noise input. A_Δ, B_Δ, C_Δ, D_Δ and L_Δ are appropriately dimensioned uncertain matrices. The parametric uncertainty of the system is modeled via an LFT structure for the time-invariant linear system S_{LFT} shown in Fig. 1. Therefore, the uncertain system can be modeled by

$$\begin{aligned} \dot{x} &= Ax + B_w w + B_p p, \\ y &= C_y x + D_{yw} w + D_{yp} p, \\ q &= C_q x + D_{qw} w + D_{qp} p, \\ p &= \Delta q, \end{aligned} \tag{2}$$

where $\Delta \in \mathbb{R}^{n_p \times n_q}$ is a time-invariant uncertain matrix with block diagonal structure belonging to the following set

$$\boldsymbol{\Delta} = \{\Delta = I_{n_q} \otimes \Theta \mid \quad \Theta \in U\},$$

where U represents the polytopic uncertainty set given as follows:

$$U = \{\Theta \mid \Theta = \sum_{i=1}^{n_v} \alpha_i \Theta_i\}, \tag{3}$$

where $\Theta_i \in \mathbb{R}^m$ is considered as the parameter vector representing the ith vertex of a polytopic domain with n_v vertices and $\alpha = [\,\alpha_1 \cdots \alpha_{n_v}\,]^T$ belongs to the unit simplex

$$\Lambda = \{\alpha \in \mathbb{R}^{n_v} \mid \sum_{i=1}^{n_v} \alpha_i = 1,\ \alpha_i \geq 0,\ i = 1, \cdots, n_v\}.$$

In this paper, we assume that the representation (1) is well-posed over $\boldsymbol{\Delta}$, meaning that $det(I - D_{qp}\Delta) \neq 0$ for all $\Delta \in \boldsymbol{\Delta}$. Additionally, it is assumed that system (1) is asymptotically stable for all $\Delta \in \boldsymbol{\Delta}$. To be concise, we adopt the notation

$$\Delta_a = \Delta(I - D_{qp}\Delta)^{-1}, \tag{4}$$

therefore, we obtain

$$\begin{aligned} A_\Delta &= A + B_p \Delta_a C_q, \\ B_\Delta &= B_w + B_p \Delta_a D_{qw}, \\ C_\Delta &= C_y + D_{yp} \Delta_a C_q, \\ D_\Delta &= D_{yw} + D_{yp} \Delta_a D_{qw}. \end{aligned} \tag{5}$$

Similarly, we assume

$$L_\Delta = L_c + L_p \Delta_a C_q, \tag{6}$$

where L_c and L_p are known constant matrices with appropriate dimensions. The objective is to estimate the signal z by a robust full-order linear filter of general structure described by:

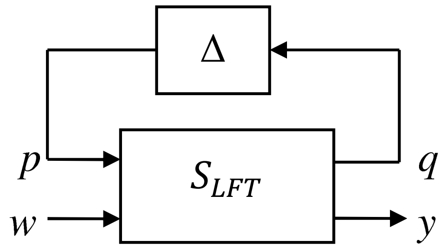

Fig. 1. LFT representation.

$$\begin{aligned} \dot{\hat{x}} &= A_F \hat{x} + B_F y, \\ \hat{z} &= C_F \hat{x} + D_F y, \end{aligned} \tag{7}$$

where $\hat{x}$ is the filter state vector and (A_f, B_f, C_f, D_f) are appropriately dimensioned filter matrices to be determined. Define estimate error $e \triangleq z - \hat{z}$. Let $\eta \triangleq [\, x^T \ \hat{x}^T \,]^T$ denote the states of the augmented system

$$\begin{aligned} \dot{\eta} &= \bar{A}\eta + \bar{B}w, \\ e &= \bar{C}\eta + \bar{D}w, \end{aligned} \tag{8}$$

where

$$\bar{A} = \begin{bmatrix} A_\Delta & 0 \\ B_F C_\Delta & A_F \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} B_\Delta \\ B_F D_\Delta \end{bmatrix},$$
$$\bar{C} = [\, L_\Delta - D_F C_\Delta \ \ -C_F \,], \quad \bar{D} = -D_F D_\Delta.$$

Let H_Δ denote the transfer function from w to e. In this paper, two filtering problems are considered as follows:

I:Robust H_2 filtering problem

Consider the uncertain system (1) where w is assumed to be a zero-mean Gaussian white noise process with identity power spectrum density matrix. Determine a robust filter, given by (7), such that $E[e^T e] < \gamma$ for all $\Delta \in \boldsymbol{\Delta}$ for the smallest possible γ. The filter is thus the solution of the following optimization problem:

$$\min_{A_f, B_f, C_f, D_f} \gamma$$
$$s.t.$$
$$\|H_\Delta\|_2^2 \triangleq \frac{1}{2\pi} \int_{-\pi}^{\pi} Tr(H_\Delta(e^{j\omega})^* H_\Delta(e^{j\omega})) d\omega < \gamma \forall \Delta \in \boldsymbol{\Delta}.$$

II. Robust H_∞ filtering problem

Consider the uncertain system (1) where w is assumed to be a bounded energy disturbance. Determine a robust filter, given by (7), such that $\|e\|_2^2 < \gamma \|w\|_2^2$ for all $\Delta \in \boldsymbol{\Delta}$ for the smallest possible γ. The filter is thus the solution of the following optimization problem:

$$\min_{A_f, B_f, C_f, D_f} \gamma$$
$$s.t.$$
$$\|H_\Delta\|_\infty^2 \triangleq \sup_{\|w\|_2^2 \neq 0} \frac{\|e\|_2^2}{\|w\|_2^2} < \gamma \quad \forall \Delta \in \boldsymbol{\Delta}.$$

for all nonzero $w \in l_2[0, \infty)$.

The following lemma is borrowed from Sadeghzadeh (2015) and required to proceed further.

Lemma 1. Let $\boldsymbol{\Sigma}$ be the set of all matrices

$$\Sigma_\tau = \begin{bmatrix} \Sigma_c & \Sigma_p \\ \Sigma_p^T & \Sigma_{pp} \end{bmatrix}, \tag{9}$$