Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-9 (2016) 175–179

Robust Fractional Order Controller for Chaotic Systems

Djalil Boudjehem * Badreddine Boudjehem **

* *Advanced Control Laboratory, Department of Electronic and Telecommunication, university of Guelma, BP 401, Guelma 24000, Algeria. Tel.: +213-37-203348, Fax: +213-37-203348 (e-mail: boudjehem.djalil@ gmail.com)*

** *Advanced Control Laboratory, Department of Automatic and Electrotechnique, university of Guelma, BP 401, Guelma 24000, Algeria. Tel.: +213-37-203348, Fax: +213-37-203348 (e-mail: bboudjehem@gmail.com)*

Abstract: In this paper, we propose a robust fractional controller structure to control chaotic systems. The fractional controller structure is based on the predictive control of chaotic systems. The parameters of the controller are obtained by minimizing the error state energy.

Simulation results on Chen and Lorenz chaotic systems show the effectiveness of the proposed controller to reject the noise and the disturbance injected to these systems states. These results show also that the proposed controller gives better results than the standard predictive control.

Keywords: Fractional calculus, predictive control, chaotic systems, noise perturbation rejection.

1. INTRODUCTION

The study of the chaotic systems have become very important in the last decade. It also had the attention of many researchers where efforts were being made in an attempt to stabilize these systems.

The first research that summarized the born of chaotic systems is the work of Edward Lorenz (Lorenz (1963)), in which he discovery the chaotic behavior in the weather. He made a presentation on the sensitivity of some systems to minor changes in the initial conditions. In (Li and Yorke (1975)), LI presented for the first time the word chaos.

After several studies on the chaotic systems, their control has become an increasing requirement, and started first by the work of Ott *et al* (Ott et al. (1990)), they present the control of dynamical systems with chaotic behavior at the first time. their technique which called later OGY method represents one of the main methods in the field. Its principle is that, we can control the chaotic system by applying small perturbation to one of the system's parameters. In the OGY method, the control will start only when the orbit trajectory is very close to the desired trajectory, which may take a while to be satisfied. The control will also take effect only around a fixed point, which may also take a long time do be realized. Other works had been proposed to control chaotic systems by using periodic perturbation methods (Lima and Pettini (1990); Azevedo and Rezende (1991); Braiman and Goldhirsch (1991)).

In (Pyragas (1992)), Pyragas proposed a method that permits to stabilize unstable periodic orbits. This method is based on adding a delayed state term, in order to force the dynamical evolution of the system to track a desired periodic dynamics. A generalized form of Pyragas method was discussed in (Bleich and Socolar (1996)).

Nakajima (Nakajima (1997)), stated that in the use of Pyragas method, the unstable periodic orbit will not be stabilized if the jacobian matrix has an odd number of real eigenvalues higher than the unity.

Ushio an Yamamoto (see Ushio and Yamamoto (1998, 1999); Yamamoto et al. (2001)), propose a predictive feedback stat controller based on the principle of Pyragas. The control signal is based on the error between the current and future state (predicted one). The controlled states will converge to the fixed points due to the absence of the approximated calculation in the feedback loop.
More recent works that uses predictive control of chaotic systems, base their works on the results of Ushio and Yamamoto (see Yoo et al. (2005); Boukabou et al. (2008); Sadaoui et al. (2011); Longge and Xiangjie (2013)).

Even though, with the good obtained results, the control of chaotic systems needs more efforts to achieve better results. This is due to the fact that chaotic systems are strongly sensitive to changes in initial conditions, the presence of noise measurements and disturbances. This fact, makes the control system unable to stabilize the chaotic systems in such cases.

Developing more robust controllers imposes the use of fractional calculus. The use of fractional calculus is widespread in the recent period, because of the best obtained results in the design of a successfully effective controllers. The

10.1016/j.ifacol.2016.07.525

ability of the fractional controllers to reject perturbations and maintain the global desired performances even with the presence of a noise and disturbance in the systems under control, makes them a subject of interest of many researchers (see Boudjehem and Boudjehem (2012); Boudjehem et al. (2013); Boudjehem and Boudjehem (2013); Li et al. (2010); Machado (2010); Jesus et al. (2010); Machado (2013))

In this paper, we propose to use a fractional order controller structure to control chaotic systems, the controller parameters will be determined by minimizing the state error energy. The obtained controller is able to maintain the states stable.

We compared the results obtained using the proposed controller and those obtained using conventional predictive control of Lorenz and Chen chaotic systems. The Simulation show that the proposed controller gives better results and ensures the best states stabilization.

2. FRACTIONAL ORDER SYSTEMS

A fractional order system is a system that is described by a differential equation with a fractional operator. In this equation the classical differential operator is replaced by a fractional one and is defined due to Riemann and Liouville fractional integral version,(Boudjehem and Boudjehem (2012); Boudjehem et al. (2013); Boudjehem and Boudjehem (2013); Machado (2013)), and it is given by 1. It is called the fractional derivative of order α_n with respect to variable t and with the starting point $t = 0$.

$$\frac{d^{\alpha_n}}{dt^{\alpha_n}} f = D^{\alpha_n} f = {}_0 D_t^{\alpha_n} f \tag{1}$$

$$D^{\nu} f(x) = \frac{1}{\Gamma(n-\nu)} \left(\frac{d}{dx}\right)^n \int_0^x (x-t)^{n-\nu-1} f(t)dt. \tag{2}$$

where n is an integer number defined as $n-1 < \nu < n$ and $\Gamma(\nu)$ is the Euler's *Gamma* function defined by: $\Gamma(\nu) = (\nu - 1)!$, with the property: $\Gamma(\nu + 1) = \nu\Gamma(\nu)$.
Taking the the Laplace transform of equation 2 we get:

$$\mathcal{L}[D^{\nu} f(x)] = s^{\nu} F(s) - \sum_{k=0}^{n-1} s^{n-k-1}\ D^{k-n+\nu} f(0) \quad \text{if } Re(\nu) > 0 \tag{3}$$

where n is defined before and if $Re(\nu) \leq 0$ the summation in the right hand side of (3) will be equals to zero.

To be able to use fractional operators in system's implementation, we need to approximate the fractional differential operator s^{ν} by using classical integer transfer functions. It may result using such approximation to get a transfer functions with infinite number of zero and poles. But is is always possible to get good approximations Oldham and Spanier (1974).
Number of approximations that are used are based on a recursive distribution of poles and zeroes (Oustaloup (1991)), and The most common one used is the approximation proposed by Oustaloup (Oustaloup (1991)):

$$s^{\nu} \approx C \prod_{k=-N}^{N} \frac{1 + (s/\omega_{z_k})}{1 + (s/\omega_{p_k})}, \quad \nu > 0. \tag{4}$$

ω_l, ω_h are the lower and higher frequency approximation interval. This means that the approximation is valid in that frequency interval. The gain C has the role of approximation tuning, so it is adjusted until both sides of (4) will have unit gain at 1 rad/s. More details on this approximation can be found in (Oustaloup (1991); Boudjehem et al. (2013)) In general, it is usual to split fractional powers of s like this:

$$s^{\nu} = s^n s^{\delta}, \quad \nu = n + \delta \tag{5}$$

where n is an integer number defined as: $n < \nu < n + 1$, thus the values of δ will be compromise between 0 and 1. Therefor we need only to approximate the latter term.

3. CONTROLLER DESIGN

The predictive control of a chaotic system described by 6 is based on introducing a control signal u (Ushio and Yamamoto (1999))(see equation 7), and u takes the form given by 8

$$\dot{X} = f(X, t) \tag{6}$$

$$\dot{X} = f(X, t) + u \tag{7}$$

$$u = K(X_{predicted} - X_t) \tag{8}$$

K is the gain to be identified and $X_{predicted}$ is the predicted state of the chaotic system.

But due to the hight sensitivity of chaotic systems to the initial conditions and to the presence on some uncertainty of one or a number of the system parameters, makes the defined controller unable to preserve the obtained stability in such cases.

Instead, we propose a fractional structure of the controller. This structure is defined by equation 9

$$u = K\ s^{\alpha}(X_{predicted} - X_t) \tag{9}$$

K and α are now the gain and fractional order of the fractional controller. To identify these parameters, we used a minimization criterion based on the energy of the state error energy.

$$J = \min_{K,\alpha} \int e^2 \tag{10}$$

e is the state error signal.

4. SIMULATION RESULTS

In this part, we apply the proposed technique to identify the parameter of the robust fractional controller used with the Chen and Lorenz chaotic systems. After that, the obtained controllers are used with the presence of a perturbation unit step signal and a random noise with a maximum amplitude equals to unity. The obtained results are compared to those obtained using a predictive control given by the following structure: