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A Stable and Robust NMPC Strategy with Reduced Models and Nonuniform Grids

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Abstract: Nonuniform discretizations of state and control profiles and model reduction are essential to approximate discretized DAE systems, capture multiple time scales of the state profiles and reduce the size of nonlinear programming (NLP) subproblems for off-line optimal control problems. These discretizations are often dictated by dynamic characteristics that depend on the system application. However, nonuniform grids in Nonlinear MPC (NMPC), which we denote as *input and state blocking strategies*, may not lead to recursive feasibility, a key property for nominal stability that follows directly with uniform grids. In this study, we analyze a class of NMPC blocking strategies and show that nominal stability and input-to-state stability (ISS) can be preserved with these formulations. These strategies are especially useful for large first principles models, as we demonstrate on a bubbling fluidized bed (BFB) process that captures CO_2 from flue gas. With this case study we demonstrate that input and state blocking, along with model reduction, leads to accurate state profiles, nominal and robust stability, far less computation, and essentially the same NMPC performance as with uniform grids.

Keywords: Nonlinear model predictive control, nonlinear programming, move blocking, robust stability, model reduction

1. INTRODUCTION

With increasing applications of NMPC, additional research is needed for the efficient solution of NLP subproblems for larger plant systems. This study explores input and state blocking along with model reduction within NMPC strategies. These features arise in the use of nonuniform grids derived from direct transcription strategies in optimal control, through high order collocation discretizations with finite element grids. The construction of these grids is tailored to the dynamics of particular applications. In this way accurate state and control profiles can be obtained efficiently, and nonuniform grids are frequently considered for *off-line* implementations of dynamic optimization. This study expands this task to on-line controllers as well.

A closely related problem to nonuniform grids for NMPC is the use of input or move blocking in MPC. This problem is widely applied in commercial implementations of DMC and other MPC controllers, through the specification of output prediction horizons and shorter horizons for manipulated variables (see, e.g, Prett and Garcia (1988)). On the other hand, when terminal costs and constraints are imposed, input blocking raises several challenges with respect to stability and robustness properties. In particular, most moving horizon input blocking (MHB) schemes are not recursively feasible and this can complicate the stability analysis.

Stability and robustness properties of various input blocking schemes have been analyzed over the past decade for linear MPC. Cagienard et al. (2007) developed a general cyclic blocking scheme based on input deviations from an unconstrained feedback controller. This blocking scheme cycles over a time period and maintains recursive feasibility, even for terminal conditions. However, the controller moves are more restricted through these input deviations and optimal performance of the blocked MPC strategy is not guaranteed. Gondhalekar and Imura (2010) establish a blocking scheme that applies to all blocking patterns by initially establishing feasibility regions for the blocked controller. Their approach then finds the least restrictive moves for a given blocking scheme, and ensures recursive feasibility, but without stability guarantees. Shekhar and Maciejowski (2012) develop a blocking framework for variable horizon MPC, which allows shifting and transformation of blocking patterns as the horizons evolve. They include a robust stability analysis using contraction properties and require terminal constraints on the MPC problem.

These studies show that an alternative shifted blocking (SB) scheme, where the left-most interval is removed and a right-most interval is added as the horizon shifts, is recursively feasible if appropriate terminal conditions are imposed. A particular case considered in Würth and Marquardt (2014) is based on approximations to infinite horizon NMPC, where a shrinking horizon is maintained over infinite time. Under these conditions approximations to MHB and SB schemes are equivalent and recursively feasible.

In the next section we describe our MHB and SB schemes for nonuniform grids and review nominal and ISS stability properties for NMPC. Based on these we modify the NLP subproblem for blocked NMPC to enforce strong descent of the Lyapunov function at each sampling time. This leads to an NMPC strategy that embeds both MHB and SB schemes and leads to robust stability guarantees. Section 3 demonstrates our blocking NMPC strategy on a bubbling fluidized bed (BFB) process, using both first principles and reduced models, along with input and state blocking. Section 4 concludes the paper and discusses areas for future work.

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2. INPUT AND STATE BLOCKING FOR NMPC

Consider the following discrete-time nonlinear dynamic model of the plant with uncertainties:

$$\begin{aligned} x(k+1) &= \hat{f}(x(k), u(k), w(k)) \\ &= f(x(k), u(k)) + d(x(k), u(k), w(k)) \end{aligned} \tag{1}$$

where $x(k) \in \Re^{n_x}$, $u(k) \in \Re^{n_u}$ and $w(k) \in \Re^{n_w}$ are the plant states, controls and disturbance signals, respectively, defined at time steps t_k with integers $k > 0$. The mapping $f : \Re^{n_x+n_u} \mapsto \Re^{n_x}$ with $f(0,0) = 0$ represents the nominal model, while the term $d : \Re^{n_x+n_u+n_w} \mapsto \Re^{n_x}$ is used to describe modeling errors, estimation errors and disturbances. We assume that $f(\cdot,\cdot)$ and $d(\cdot,\cdot,\cdot)$ are Lipschitz continuous, and that the noise $w(k)$ is drawn from a bounded set $\mathscr{W}$.

We also consider a blocking pattern $\mathbf{v} = Mq$ where $\mathbf{v} = [v_0^T, v_1^T, \ldots, v_{N-1}^T]^T$ and q are the blocked inputs. After N_0 intervals the blocking M matrix incorporates n_b blocks, each of length $N_j, j = 1, \ldots, n_b$ as follows:

$$M = \begin{bmatrix} I_{n_u \times N_0} & 0 & 0 & \ldots & 0 \\ 0 & E_1 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & 0 & \ldots & E_{n_b} \end{bmatrix} \tag{2}$$

where the matrices $E_j, j = 1, \ldots n_b$ consist of N_j stacked identity matrices of order n_u.

Our blocked nonlinear model predictive controller (NMPC) is defined over a horizon where $\sum_{j'=0}^{n_b} N_{j'} = N$. We assume that the states and controls are restricted to the domains $\mathbb{X}$ and $\mathbb{U}$, respectively. $\mathbb{X}_f$ is the terminal set with $\mathbb{X}_f \subset \mathbb{X}$. The set $\mathbb{U}$ is compact and contains the origin; the sets $\mathbb{X}$ and $\mathbb{X}_f$ are closed and contain the origin in their interiors. We consider a stage cost given by $\psi(\cdot,\cdot) : \Re^{n_x+n_u} \to \Re$, while the terminal cost is denoted by $\Psi(\cdot) : \Re^{n_x} \to \Re$; both are assumed to be Lipschitz continuous. Moreover, we apply the robust problem formulation in Yang et al. (2015) and relax $\mathbb{X}$ and $\mathbb{X}_f$ with ℓ_1 penalty terms, with a sufficiently large penalty parameter ν. Writing $\mathbb{X}$ and $\mathbb{X}_f$ as inequalities $g(z_l) \le 0$ and $g(z_N) \le 0$, respectively, and redefining $g_+^{(j)}(z_l) = max(0, g^{(j)}(z_l))$, $\psi(z_l, v_l) := \psi(z_l, v_l) + \nu \|g_+(z_l)\|$ and $\Psi(z_N) := \Psi(z_N) + \nu \|g_+(z_N)\|$, we obtain the following MHB reformulation:

$$V(x(k)) := \min_{v_l, z_l} \Psi(z_N) + \sum_{l=0}^{N-1} \psi(z_l, v_l) \tag{3}$$

$$\text{s.t.} \quad z_{l+1} = f(z_l, v_l), l = 0, \ldots N_0 - 1$$

$$z_{l+1} = f^j(z_l, v_l),\ l = \sum_{j'=0}^{j-1} N_{j'}, \ldots, \sum_{j'=0}^{j} N_{j'} - 1, j = 1, \ldots, n_b$$

$$z_0 = x(k), \mathbf{v} = Mq, v_l \in \mathbb{U}.$$

Note that the redefined objective function in (3) is no longer differentiable everywhere, but still Lipschitz continuous, with Lipschitz constant L_V, which is sufficient for the stability analysis in Section 2.1.

In (3) we note that a coarser approximation is allowed for the differential-algebraic model, $z_{l+1} = f^j(z_l, v_l)$. This model leads to state profiles described by finite elements of different lengths in each block. The longer elements are sufficient for slower time scales and lead to a significant reduction in NLP variables. The accuracy of these models is based on problem dependent features where the blocking pattern, $N_j, j = 1, \ldots n_b$ is determined by the system dynamics. As a result the blocked state model (based on collocation on nonuniform finite elements) is assumed to be a high fidelity approximation to the plant.

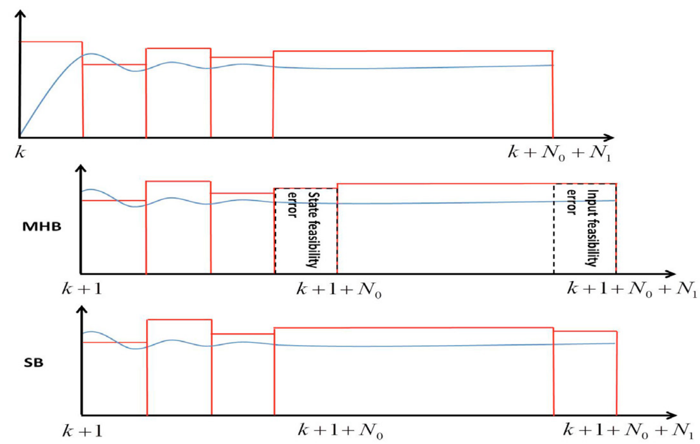

Figure 1. Representation of MHB and SB schemes

Figure 1 gives a graphic representation of state and input profiles in the proposed blocking schemes. From Figure 1 we can see that MHB is not recursively feasible, neither for input nor state blocking. Instead, as described in Section 2.2, the state and input feasibility error in MHB depends on choice of the blocking pattern and treatment of model mismatch. Moreover, as shown in Figure 1, we also consider a Shifted Blocking (SB) strategy where $\mathbf{v} = \bar{M}\bar{q}$ and

$$\bar{M} = \begin{bmatrix} I_{n_u \times (N_0-1)} & 0 & 0 & \ldots & 0 \\ 0 & E_1 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & E_{N_b} & 0 \\ 0 & 0 & \ldots & 0 & I_{n_u} \end{bmatrix}. \tag{4}$$

The SB pattern modifies the MHB pattern by removing the sampling time on the left and adding a sampling interval on the right. The SB strategy is recursively feasible for the inputs and states, as well as the terminal conditions. The NLP subproblem for the SB pattern is given by:

$$\bar{V}(x(k)) := \min_{v_l, z_l} \Psi(z_N) + \sum_{l=0}^{N-1} \psi(z_l, v_l) \tag{5}$$

$$\text{s.t.} \quad z_{l+1} = f(z_l, v_l), l = 0, \ldots N_0 - 2$$

$$z_{l+1} = f^j(z_l, v_l), l = (\sum_{j'=0}^{j-1} N_{j'}) - 1, \ldots (\sum_{j'=0}^{j} N_{j'}) - 2, j = 1, \ldots n_b$$

$$z_N = f(z_{N-1}, v_{N-1}), z_0 = x(k), \mathbf{v} = \bar{M}\bar{q}, v_l \in \mathbb{U}$$

2.1 Nominal and ISS Stability Properties

Stability properties of blocked NMPC are adapted from well-known properties of the standard NMPC controller (Magni and Scattolini (2007); Keerthi and Gilbert (1988)), with the following assumptions:

Assumption 1. (Nominal Stability Assumptions for NMPC)

- The terminal penalty $\Psi(\cdot)$, satisfies $\Psi(z) > 0, \forall z \in \mathbb{X}_f \backslash \{0\}$,
- There exists a local control law $u = \kappa_f(z)$ defined on $\mathbb{X}_f$, such that $f(z, \kappa_f(z)) \in \mathbb{X}_f, \forall z \in \mathbb{X}_f$, and $\Psi(f(z, \kappa_f(z))) - \Psi(z) \le -\psi(z, \kappa_f(z)), \forall z \in \mathbb{X}_f$.