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Robust optimization of water-flooding in oil reservoirs using risk management tools

M.Mohsin Siraj *, **Paul M.J. Van den Hof** * and **Jan Dirk Jansen** **

* *Electrical Engineering, Control Systems group, Eindhoven University of Technology (e-mails: m.m.siraj, p.m.j.vandenhof@tue.nl).*
** *Department of Geoscience and Engineering, Delft University of Technology, The Netherlands. (e-mail: j.d.Jansen@tudelft.nl)*

Abstract: The theory of risk provides a systematic approach to handling uncertainty with well-defined risk and deviation measures. As the model-based economic optimization of the water-flooding process in oil reservoirs suffers from high levels of uncertainty, the concepts from the theory of risk are highly relevant. In this paper, the main focus is to offer an asymmetric risk management, i.e., to maximize the lower tail (worst cases) of the economic objective function distribution without heavily compromising the upper tail (best cases). Worst-case robust optimization and Conditional Value-at-Risk (CVaR) risk measures are considered with geological uncertainty to improve the worst case(s). Furthermore, a deviation measure, semi-variance, is also used with both geological and economic uncertainty to maximize the lower tail. The geological uncertainty is characterized by an ensemble of geological model realizations and the economic uncertainty is defined by an ensemble of varying oil price scenarios.

Keywords: Water-flooding optimization, Theory of risk, Handling uncertainty, Risk measures

1. INTRODUCTION

Risk is a broad concept covering different social and human sciences, e.g., ethics, psychology, medicine, economics etc. As a general definition, risk is an unexpected result or the probability of a failure. Theory of risk helps in modeling (or defining) risk, measuring it, and also provides tools to minimize or manage it, see e.g., Artzner et al. (1999), Krokhmal et al. (2011). From a financial viewpoint, risk can be defined as the unpredicted variability or a potential loss of the expected economic objective. Markowitz Markowitz (1952) in the early 50's has proposed a 'risk-return' portfolio selection approach, where the risk is characterized as the variance of the individual assets.

In the oil reservoir water-flooding optimization, a financial objective, e.g., Net Present Value (NPV) is maximized, see e.g., Brouwer and Jansen (2004), Foss (2012) and Van den Hof et al. (2012). Due to the limited knowledge of the reservoir model parameters and the varying economic conditions, this model-based economic optimization suffers from high levels of uncertainty. As risk management plays an important role in decision making under uncertainty (Rockafellar (2007)), water-flooding optimization becomes a natural candidate to use concepts from the theory of risk. In the petroleum engineering literature, decision making under uncertainty has been discussed from various perspectives. In Van Essen et al. (2009), a so-called robust optimization approach has been introduced, which maximizes an average NPV over an ensemble of geological model realizations. In Capolei et al. (2015b), a symmetric mean-variance optimization approach has been implemented honoring geological uncertainty. In Siraj et al. (2015a), these approaches have been extended to consider the economic uncertainty characterized by varying oil price scenarios. Similar strategies have been described in Yeten et al. (2003), Bailey et al. (2005) and Yasari et al. (2013). One of the main limitations of the mean-variance optimization approach is the symmetric nature of the variance which also penalizes the best cases, while generally, in a maximization problem, the decision maker is mainly concerned with the lower tail of the objective function distribution. In Xin and Albert (2015), a multi-objective optimization has been implemented that maximize the average of the objective function and the worst case with respect to the geological uncertainty. As an early work of using the theory of risk in water-flooding optimization, different risk measures with their pros and cons have been reviewed in Capolei et al. (2015a) and their suitability for the production optimization is studied. In Siraj et al. (2015b), asymmetric risk measures have been studied and implemented with economic uncertainty.

The main contribution of this work is to address the question of how the well-defined risk and deviation measures in the theory of risk can be beneficial in providing an asymmetric risk management of the objective function, i.e., NPV distribution? Both geological and economic uncertainties are considered. The asymmetric risk measures such as the worst-case max-min approach (Bertsimas et al. (2011)) and the Conditional Value-at-Risk (CVaR) (Rockafellar and Uryasev (2000)) are implemented with geological uncertainty characterized by an ensemble of reservoir models. The worst-case approach, that maximizes the worst-case in a given uncertainty set, and the CVaR, defined as the average of some percentage of the worst-case scenarios, allow for an asymmetric shaping of the objective

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function distribution. The asymmetric deviation measure, semi-variance, originally proposed in Markowitz (1952), provides a measure for the return being below the expected return. It is also considered and implemented with both geological and economic uncertainties. The economic uncertainty is characterized by an ensemble of varying oil prices.

The paper is organized as follows: In the next section, uncertainty in model-based economic optimization of the water-flooding process is explained. In Section 3, the worst-case optimization approach is presented with a simulation example. CVaR optimization with a simulation example under geological uncertainty is presented in Section 4. Section 5 discusses the Semi-variance approach in detail with a simulation example with both geological and economic uncertainty. Finally the conclusions of the presented results are given in Section 6.

2. UNCERTAINTY IN WATER-FLOODING OPTIMIZATION

Water-flooding involves the injection of water in an oil reservoir to increase oil production. NPV, as an objective for the dynamic optimization of the water-flooding process, can be mathematically represented in the usual fashion as:

$$J = \sum_{k=1}^{K} \left[\frac{r_o \cdot q_{o,k} - r_w \cdot q_{w,k} - r_{inj} \cdot q_{inj,k}}{(1+b)^{\frac{t_k}{\tau_t}}} \cdot \Delta t_k \right] \quad (1)$$

where r_o, r_w and r_{inj} are the oil price, the water production cost and the water injection cost in $\frac{\$}{m^3}$ respectively. K represents the production life-cycle i.e., the total number of time steps k and Δt_k the time interval of time step k in days. The term b is the discount rate for a certain reference time τ_t. The terms $q_{o,k}, q_{w,k}$ and $q_{inj,k}$ represent the total flow rate of produced oil, produced water and injected water at time step k in $\frac{m^3}{day}$.

The limited information contents in seismic, well logs and production data about the true reservoir parameters result in highly uncertain reservoir models. Similarly, the NPV objective function contains economic variables such as interest rate, oil price, etc., which fluctuate with time and can not be precisely predicted. The first step in handling uncertainty is the modeling (quantification) of the uncertainty space Θ. A general practice of quantifying uncertainty in the water-flooding optimization is by considering an ensemble of uncertain parameters, see e.g., Van Essen et al. (2009), Capolei et al. (2015b). It is equivalent to descretizing the uncertainty space, i.e., $\Theta_{N_{geo}} := \{\theta_1, \theta_2, \cdots, \theta_{N_{geo}}\}$, where θ_i is a realization of uncertain parameter in an ensemble of N_{geo} members.

Water-flooding optimization is a highly complex large-scale non-linear optimization problem. In this work, a gradient-based optimization approach is used where the gradients are obtained by solving a system of adjoint equations, see e.g., Jansen (2011). An optimization solver KNITRO (Byrd et al. (2006)) is then used with an interior point method to iteratively converge to a (possibly local) optimum.

In the next sections, various risk/deviation measures, i.e., worst-case, CVaR and semi-variance are discussed in details with simulation examples.

3. WORST-CASE ROBUST OPTIMIZATION

Worst-case robust optimization (WCO) assumes that the uncertainty is known only within certain bounds, i.e., uncertainty set Θ, and the solution is robust for any realization of the uncertainty in the given set. Hence it focuses only on the worst-case in Θ and solves a max-min (or min-max) problem. The worst-case or a max-min optimization objective can be written as:

$$\max_{\mathbf{u}} \min_{\theta_i} J_i(\mathbf{u}, \theta_i) \quad (2)$$

where $\mathbf{u}$ is control input and $\theta_i \in \Theta_{N_{geo}}$ is the uncertain parameter. It can easily be seen that the above optimization problem is non-differentiable, so a common approach to reformulate the above *max-min* problem is by adding a slack variable z with additional constraints as follows: (Ben-Tal et al. (2009))

$$\begin{aligned} \max_{\mathbf{u},z} \quad & z \\ \text{s.t.} \quad & z \leq J_i(\mathbf{u}, \theta_i) \quad \forall i. \end{aligned} \quad (3)$$

Therefore, for a total number of ensemble members N_{geo}, there will be N_{geo} additional constraints. As the worst-case optimization only focuses on the lowest value of the NPV distribution, it provides an asymmetric risk management. The main limitation of the worst-case approach is that it provides a very conservative solution. In the next subsection, the simulation example for the worst-case approach (3) implemented with geological uncertainty is presented.

3.1 Simulation example under geological uncertainty

Simulation tools: All the simulation experiments in this work are performed using MATLAB Reservoir Simulation Toolbox (MRST) (Lie et al. (2012)) while KNITRO (Byrd et al. (2006)) is used for subsequent optimization.

Reservoir models: We use an ensemble of $N_{geo} = 100$ geological realizations of the Standard egg model, see Jansen et al. (2014). Each model is a three-dimensional realization of a channelized reservoir produced under water flooding conditions with eight water injectors and four producers based on the original Egg model proposed in Van Essen et al. (2009). The true permeability field is considered to be the only unknown parameter and the number of 100 realizations is assumed to be large enough to be a good representation of this parametric uncertainty space. The life-cycle of each reservoir model is 3600 days. The absolute-permeability field of the first realization in the set is shown in Fig. 1. Fig. 2 shows the

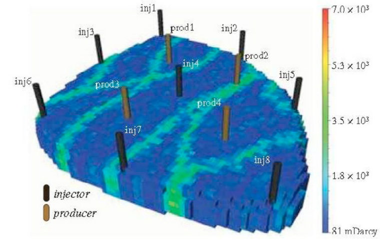

Fig. 1. Permeability field of realization 01 of a set of 100 realizations

permeability fields of six randomly chosen realizations of