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On the significance of the noise model for the performance of a linear MPC in closed-loop operation

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Abstract: This paper discusses the significance of the noise model for the performance of a Model Predictive Controller when operating in closed-loop. The process model is parametrized as a continuous-time (CT) model and the relevant sampled-data filtering and control algorithms are developed. Using CT models typically means less parameters to identify. Systematic tuning of such controllers is discussed. Simulation studies are conducted for linear time-invariant systems showing that choosing a noise model of low order is beneficial for closed-loop performance.

Keywords: Closed-loop control, predictive control, model-based control, Kalman Filters, state estimation

1. INTRODUCTION

Model Predictive Control (MPC) is a control methodology that uses a model of the system to be controlled to predict its output over a future horizon. At each time instance a control sequence is calculated online as the solution to an open-loop control problem based on the model, the current state and specified reference trajectory. Only the first element of the control sequence is applied to the system and feedback is obtained by repeating this procedure when the next measurements are received. A notable advantage of MPC is the way constraints are handled directly when solving the optimization problem resulting in the control sequence. The performance of the controller thus hinges on the quality of the system model, but not only on that. Also noise and possible disturbances must be catered for, see Pannocchia and Rawlings (2003), Gopaluni et al. (2004), Gevers (2005), Shah and Engell (2010), Huusom et al. (2012) and references therein. One should therefore consider a system model comprising a deterministic as well as a stochastic or noise part. Selecting a noise model involves a trade-off between conflicting requirements namely those of low variance set-point tracking, disturbance rejection and fast response to unmeasured disturbances.

Boiroux et al. (2015) provided a comparative study of the effects of choosing different deterministic model parts in MPC-based Artificial Pancreas technology keeping the stochastic part fixed. The goal of the present paper is to study the role played by the stochastic part of the model. This term is intended to absorb not only the presence of unmeasured disturbances but also more generally unmodelled system dynamics. The ultimate test of the suitability of a given noise model is therefore whether the system performs adequately in closed-loop (CL). The message of this paper is that closed-loop performance may benefit from selecting a suitable low-order noise model.

CL performance is evaluated for model structures corresponding to different filter orders. For each model structure the noise term is identified using the Maximum Likelihood (ML) criterion from measurements collected before closing the loop, see Jørgensen and Jørgensen (2007b). An efficient MPC implementation is developed based on continuous-time transfer functions keeping the deterministic and stochastic model parts separate. The stochastic part will determine the Kalman Filter and Predictor while the deterministic model part, set-point and filtered state estimates will determine the optimal control problem to be solved.

The paper is structured as follows. Section 2 recalls the basic theory of realization and discretization of linear time-invariant (LTI) systems given in terms of transfer functions. The following section continues by focusing on the case of LTI systems with continuous-time white noise input. Section 4 develops the Kalman Filter and Predictor for the resulting discrete-time state space model and is followed by a section developing the Model Predictive Controller. We round off with a section discussing the outcome of a concrete closed-loop control simulation for noise models of different orders.

2. REALIZATION OF LINEAR SYSTEMS

We consider a linear system described in continuous time in terms of transfer functions $G(s)$ and $H(s)$ and with discrete measurements $y(t_k)$ at times $t = t_k$:

$$\begin{aligned} Z(s) &= G(s)\,U(s) + H(s)\,W(s) \\ y(t_k) &= z(t_k) + v_k, \qquad k = 0, 1, 2, ... \end{aligned} \tag{1}$$

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Here, U denotes the input to the deterministic part of the model and W the white noise input to the stochastic part of the model. We shall assume that G and H are strictly proper. Finally, $\{v_k\} \sim N_{iid}(0, r^2)$ is a sequence of independent and identically distributed Gaussian random variables representing the measurement noise. With a view to using (1) as a model of a system to be subjected to Model Predictive Control we now turn to realizing it as a discrete-time state-space model. In doing so we consider the deterministic and stochastic parts separately and assume that the Zero-Order-Hold (ZOH) condition applies to the deterministic part. The stochastic part on the other hand involves sampling a certain Stochastic Differential Equation (SDE). We deal with this in section 3. We rely on the following lemma:

Lemma 1. Let a continuous-time system $\mathcal{S}$ be described by $Z(s) = G(s)U(s)$ where $G(s)$ is assumed to be a *proper* transfer function. When $\mathcal{S}$ is subjetcted to ZOH-input then there exist matrices A, B, C and D such that the state space model

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k \\ z_k &= Cx_k + Du_k \end{aligned} \tag{2}$$

provides a realization of $\mathcal{S}$ in discrete-time, when equidistantly sampled. If $G(s)$ is *strictly proper* we have that $D = 0$.

The deterministic part of the system description

$$Z_d(s) = G(s)\,U(s) \tag{3}$$

may be realized as a state space model by

$$Z_d(s) \sim \begin{cases} x_{k+1}^d &= A_d x_k^d + B_d u_k \\ z_k^d &= C_d x_k^d \end{cases} \tag{4}$$

and the stochastic part

$$Z_s(s) = H(s)\,W(s) \tag{5}$$

as

$$Z_s(s) \sim \begin{cases} x_{k+1}^s &= A_s x_k^s + B_s w_k \\ z_k^s &= C_s x_k^s \end{cases} \tag{6}$$

Using that

$$Z(s) = Z_d(s) + Z_s(s) \tag{7}$$

we find that there exist matrices A, B, C, G expressible in terms of the system matrices of (4) and (6) such that

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + Gw_k \\ z_k &= Cx_k \\ y_k &= z_k + v_k \end{aligned} \tag{8}$$

provides a state space realization of (1) with an added equation accounting for measurement noise v_k. In fact (8) results by taking

$$x_k = \begin{bmatrix} x_k^d \\ x_k^s \end{bmatrix} \quad A = \begin{bmatrix} A_d & 0 \\ 0 & A_s \end{bmatrix} \quad B = \begin{bmatrix} B_d \\ 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 \\ B_s \end{bmatrix} \quad C = [C_d \; C_s]$$

The process noise, $\{w_k\}$, and the measurement noise, $\{v_k\}$ are assumed to be sequences of Gaussian random variables with the joint distribution of (w_k, v_k) given by

$$\begin{bmatrix} w_k \\ v_k \end{bmatrix} \sim N_{iid}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} Q & 0 \\ 0 & R \end{bmatrix} \right) \tag{9}$$

3. REALIZATION OF STOCHASTIC TRANSFER FUNCTIONS

We now consider a continuous-time LTI system with transfer function $H(s)$ subjected to continuous-time white noise input given in the Laplace domain by $W(s)$:

$$Z(s) = H(s)W(s) \tag{10}$$

Assuming equidistant sampling with sampling time T_s the transfer function and the associated measurement equation may be realized in the form of an SDE in the sense of Itō

$$d\boldsymbol{x}(t) = A_s^c \boldsymbol{x}(t)dt + B_s^c d\boldsymbol{\omega}(t) \tag{11a}$$

$$\boldsymbol{y}(t_k) = C_s^c \boldsymbol{x}(t_k) + \boldsymbol{v}(t_k) \tag{11b}$$

where $\boldsymbol{\omega}$ denotes standard Brownian Motion and

$$\boldsymbol{x}(t_0) \sim N(\bar{x}_0, P_0) \tag{12a}$$

$$d\boldsymbol{\omega}(t) \sim N_{iid}(0, I dt) \tag{12b}$$

$$\boldsymbol{v}(t_k) \sim N_{iid}(0, R) \tag{12c}$$

with $R = r^2$.

We now discretize assuming equidistant sampling at integer multiples of T_s and obtain a discrete-time state space model (6) by taking

$$A_s = e^{A_s^c T_s} \qquad B_s = I \qquad C_s = C_s^c \tag{13}$$

and

$$w_k \sim N_{iid}(0, Q) \tag{14}$$

with

$$Q = \int_0^{T_s} e^{A_s^c \sigma} B_s^c (B_s^c)' e^{(A_s^c)' \sigma} d\sigma \tag{15}$$

The reader is referred to Åström (1970) for a proof of these discretization results. Since (A_s^c, B_s^c) is controllable it follows from Zhou et al. (1995) that Q is positive definite. According to Van Loan (1978)

$$Q = \Phi_{22}' \Phi_{12} \tag{16}$$

where

$$\exp\left(\begin{bmatrix} -A_s^c & B_s^c\,(B_s^c)' \\ 0 & (A_s^c)' \end{bmatrix} T_s \right) = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ 0 & \Phi_{22} \end{bmatrix} \tag{17}$$

The formula (17) may be employed to calculate A_s and Q numerically by means of Padé approximation, but it is in fact possible to calculate exact analytical expressions for those matrices in the case of the simple transfer functions we consider here. The expressions, however, quickly become rather unwieldy with increasing order. We introduce the notation

$$\beta = \frac{T_s}{\tau} \tag{18}$$

and list values of A_s and Q in Table 1 for examples of the kind of transfer functions considered in this paper. We note that with $H(s) = \frac{k}{(\tau s+1)^n}$ the continuous-time noise output power for white noise input derived from standard Brownian Motion becomes

$$\frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{k^2}{(1 + (\tau\omega)^2)^n} \, d\omega = \frac{1}{2\pi} \frac{k^2}{\tau} I_n \tag{19}$$

where $I_n = \int_{-\infty}^{+\infty} \frac{1}{(1+t^2)^n} \, dt$ satisfies $I_1 = \pi$ and the recursion $I_{n+1} = \frac{2n-1}{2n} I_n$ holds for $n \geq 1$. For fixed filter order n the continuous-time noise output power depends only on the ratio $\frac{k^2}{\tau}$ but the distribution of this power over the spectrum is depends on τ, spreading out more as