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Spline Fluid Models for Optimization

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Abstract: Oil production optimization is usually formulated by applying mass and momentum balances of the production network. By including temperature as a variable in pipe pressure drop, pump, and compressor models one may improve their accuracy, as well as the accuracy of the overall production system model. In addition, it is sometimes desirable to add temperature constraints to avoid flow assurance issues (*e.g.* wax and gas hydrates). The introduction of temperatures in the optimization problem requires thermodynamic properties of the fluid as functions of pressure and temperature. In this paper, a unifying fluid model for optimization using B-splines is presented. The fluid model can be constructed based on a Black-Oil model or from PVT simulations. The B-spline has properties that make it suitable for optimization. The applicability of the method is demonstrated in two examples, and the results are compared with realistic Olga simulator output.

Keywords: Energy balance, petroleum, multiphase flow, surrogate model, flow assurance

1. INTRODUCTION

Model-based approaches are increasingly used to improve the economic profit and safety of the operation in subsea oil and gas production; several testimonials to this can be found in the literature, cf. (Stenhouse, 2008; Foss, 2012).

The mass and momentum balances in the oil production network have been modeled in several works (Gunnerud and Foss, 2010; Codas and Camponogara, 2012). In the recent work by Grimstad et al. (2015a), B-spline surrogate models were used to model the nonlinear relationship between flow rates, pressure, and temperatures in multiphase pipe flow. This work did include temperature drop models, but assumed constant fluid properties. To extend this work, we propose to also model the fluid characteristics with B-splines.

In this work, we build a fluid model by fitting splines to a PVT table or black-oil model of the fluid. The splines approximate fluid properties such as the gas mass fraction and densities. The splines are nonlinear functions of the pressure and temperature that may act as surrogates for PVT simulations. When the underlying fluid model is a black-oil model, splines are used also to represent the gas compressibility factor, gas heat capacity, effective molecular weight of oil and the bubble point factor of the Lasater correlation.

An advantage with using splines is that, regardless of what the underlying fluid model is (PVT or black-oil), the resulting splines will be smooth functions with analytical derivatives readily available (Piegl and Tiller, 1997). Note that the B-spline model is nonparametric; consequently, a new B-spline must be constructed if conditions such as the oil composition were to change.

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To display the versatility of the spline fluid models we consider two modeling applications. First, we consider the modeling of subsea manifolds (commingled flows). Our objective is to find the temperature of the outlet stream from the flow rates and the temperatures of inlet streams. For this, we use the energy balance of mixing processes based on the inflow and outflow of the enthalpy.

Next, we develop a model for the heat transfer from the three-phase petroleum fluid into the environment through the pipe walls. Since the pipe properties and ambient conditions change slowly with time, we apply a steady-state model for the heat transfer. However, calculating the heat transfer coefficients (conductive and convective) requires extensive modeling efforts; it requires the consideration of pipe geometry, insulations and ambient fluid properties. Instead, we obtain the model from input-output data where the cubic B-splines are used for this modeling.

This paper is organized as follows. The spline approximation theory is described in Section 2. Then, this theory is used to define the fluid models in Section 3. Two applications of the fluid models are presented in Sections 4 and 5 which are basis for the energy balance model. The energy balance model is tested in two case studies in Section 6. Finally, the concluding remarks are given in Section 7.

2. SPLINE SURROGATE MODELS

A surrogate model can be written as

$$\Phi(x) = \sum_{i=1}^{N} c_i \phi_i(x), \tag{1}$$

where c_i are the coefficients and ϕ_i are the basis functions. Remark that the surrogate model is *linear* wrt. the coefficients; the basis functions are in general nonlinear in $x \in \mathbb{R}^n$, making Φ a nonlinear function wrt. x. The choice

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of basis functions determines the characteristics of Φ. For example, when ϕ_i are B-spline basis functions (piecewise polynomials), Φ becomes a spline function. Another class of basis functions are the radial basis functions. For such cases, Φ sometimes is referred to as a radial basis function network. An example of the latter is the *thin-plate spline*: $\phi_i(x) = ||x - x_i||_2^2 \ln(||x - x_i||_2)$, where x_i is a fixed point.

Next, consider the process of constructing of a surrogate model $\Phi : \mathbb{R}^n \to \mathbb{R}$ that approximates a (possibly unknown) function $\mathcal{F} : \mathbb{R}^n \to \mathbb{R}$.

(1) Sampling of the target function $\mathcal{F}$
(2) Fitting the surrogate model Φ to the data samples
(3) Assessing the approximation error $||\mathcal{F} - \Phi||$
    (a) If error is acceptable: stop
    (b) Otherwise: go to Step 1

An important trade-off related to this process is that of achieving a low approximation error (which may require many sample points) with few sample points (in cases where $\mathcal{F}$ is expensive to evaluate). Furthermore, if the evaluation of $\mathcal{F}$ contains random noise, care must be taken to avoid overfitting the data. A general rule of thumb – based on the principle of *Occam's razor* – is to select among models with acceptable approximation error, the one with the fewest basis functions.

Consider Step 1: Sampling of $\mathcal{F}$. We denote the set of sample points $\{x_i, y_i\}$ for $i = 1, \ldots, M$ and collect the outputs in the vector $y = [y_i]_{i=1}^M$.

In Step 2 we use the sample points to build the surrogate model. To do this, we construct the matrix $A \in \mathbb{R}^{M \times N}$ by evaluating the basis functions, so that $A_{ij} = \phi_j(x_i)$. Since the surrogate model is linear in the coefficients, the problem of fitting Φ to the data can be written as the following least-squares problem:

$$\min_c ||Ac - y||_2^2, \tag{2}$$

where the variables are the coefficients $c = [c_i]_{i=1}^N$. Assuming that we have at least as many samples as basis functions ($M \geq N$), the general solution to (2) is $c^* = (A^\mathsf{T} A)^{-1} A^\mathsf{T} y = A^\dagger y$, where $A^\dagger$ denotes the Moore–Penrose pseudoinverse.

In the special case where $M = N$, $||Ac^* - y|| = 0$ since it is possible to select the coefficients so that all sample points are interpolated. This illustrates that it is not sufficient to assess the approximation error from the value of the least-squares objective function alone. Additional sampling is required to gauge how well Φ fits the $\mathcal{F}$. This is done in Step 3 of the surrogate building process. It is also worth noting that the objective in (2) may be augmented with a regularization term to combat overfitting.

In this work we have favoured cubic B-spline basis functions, yielding a cubic B-spline surrogate model. The cubic B-spline can be constructed to obtain a high degree of smoothness – under mild assumptions it yields the interpolant in C^2 that minimizes the second-order derivative (Piegl and Tiller, 1997). Furthermore, there are fast and numerically stable algorithms for evaluating the B-spline and its derivatives, which make them suitable for optimization, as advocated by Grimstad et al. (2015a).

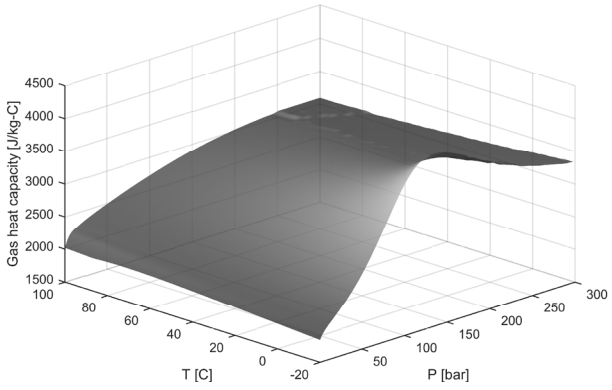

Fig. 1. Spline model for gas heat capacity, $\Phi_c^g(P, T)$

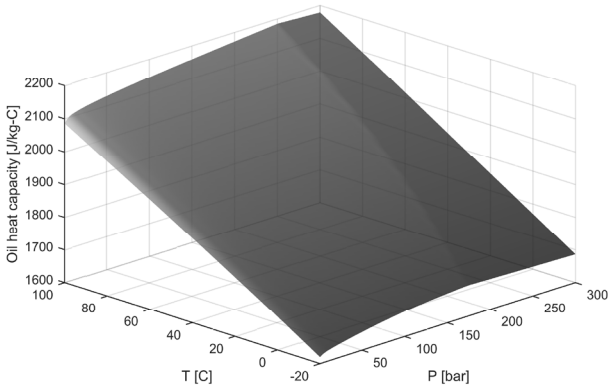

Fig. 2. Spline model for oil heat capacity, $\Phi_c^o(P, T)$

3. FLUID MODELS

Petroleum is a volatile mixture of different hydrocarbons; it expands in low pressures and light hydrocarbons (C1-C4) leave the liquid phase. Therefore, the fluid properties change for different pressures and temperatures. The gas mass fraction and other fluid properties (*e.g.* viscosity and density) at different pressures and temperature conditions can be obtained from PVT simulations or a Black Oil Model (Ahmed, 2010).

The composition of the petroleum is defined by the mole fraction or mass fraction of different light and heavy hydrocarbons, ranging from C1 to C40 or even heavier components. The composition must be known in order to perform a PVT simulation. PVTsim® is a widely used commercial package for this purpose. The results of the PVT simulation from PVTsim® are exported in tables and used as input data for flow simulators such as OLGA®. In the PVT tables, each data point corresponds to a specific pair of pressure and temperature.

In this work, we fit cubic splines to the PVT tables, following the model building process in Section 2. This yields nonlinear approximations in pressure and temperature, referred to as spline surrogate models. The fluid properties used in the proposed energy balance model are listed in Table 1. The spline models for the gas heat capacity and the oil heat capacity are shown Figures 1 and 2.