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Closed-loop Formulation for Nonlinear Dynamic Real-time Optimization ⋆

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Abstract: Dynamic real-time optimization (DRTO) is a higher level online strategy that exploits plant economic potential by making appropriate adjustments to the lower level controller set-point trajectories. In this work, we propose a closed-loop formulation for a nonlinear DRTO calculation in the form of a bilevel programming problem. A nonlinear differential algebraic equation (DAE) system that describes the process dynamic behavior is utilized with an embedded constrained predictive control (MPC) optimization subproblem to generate the approximate closed-loop response dynamics at the primary economic optimization layer. The bilevel DRTO problem is reformulated as a single-level mathematical program with complementarity constraints (MPCC) by replacing the MPC optimization subproblem by its Karush-Kuhn-Tucker (KKT) optimality conditions. We investigate the economics and control performance of the proposed strategy based on a polymer grade transition case study in the presence of plant-model mismatch and a disturbance, and a comparison is made with the application of a linear DRTO prediction model.

Keywords: real-time optimization, economic optimization, model predictive control, complementarity constraints, back-off mechanism.

1. INTRODUCTION

Process plants operate in an ever increasing environment of uncertainty and changing conditions, driven by factors such as increased global competition, variation in utility costs, restrictive environmental regulations, changing raw material prices, varying product quality specifications, and volatile market demands. Real-time optimization (RTO) is a closed-loop economic optimizer in the process automation architecture that computes set-point targets for the lower level regulatory control systems (Darby et al., 2011). The traditional RTO strategy is designed based on a steady-state model, which suffers from a limited execution frequency resulting in suboptimal operation for processes with frequent transitions and long transient dynamics. Recent advances have transformed the steady-state RTO to dynamic real-time optimization (DRTO) based on a dynamic prediction model, hence allowing process transient economics be evaluated at a substantially higher frequency.

Proposed DRTO strategies that follow a two-layer architecture generally perform economic optimization in an open-loop fashion without taking into account the presence of the plant control system, which we denote here as an open-loop DRTO strategy. In this approach, the set-points prescribed to the underlying control system are based on the optimal open-loop trajectories under an expectation that the closed-loop response dynamics at the plant level will follow the economically optimal trajectories obtained at the DRTO level. Tosukhowong et al. (2004) design the DRTO framework based on a linear(ized) process model while Würth et al. (2011) utilize a nonlinear dynamic model. An alternative to the multilevel configuration is a single-level, economic model predictive control (EMPC) approach that optimizes the plant economics at the controller sampling frequency. Such a strategy aims to address the issues of model inconsistency and conflicting objectives between the traditional RTO system and the MPC control layer, and is usually designed based on the nonlinear dynamic model describing the process behavior. In this case, the objective function could be based purely on economics (Amrit et al., 2013), or a hybrid between cost and control performance (Ellis and Christofides, 2014).

In previous work, we proposed a closed-loop DRTO strategy with a rigorous inclusion of the future MPC control calculations, which for constrained MPC cannot be expressed as an explicit continuous function to be readily included in the optimization problem. The overall closed-loop DRTO problem structure is in the form of a multilevel dynamic optimization problem with embedded MPC optimization subproblems, as illustrated in Fig. 1. It optimizes the closed-loop response dynamics of the process where the optimal control inputs are computed by a sequence of inner MPC optimization subproblems. This scheme computes the MPC set-point trajectories that determine the best economics of the predicted closed-loop response, under the assumption that the process follows the trajectory calculated by MPC until the next DRTO execution. The closed-loop DRTO formulation may be viewed as an

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EMPC approach due to explicit consideration of control performance while making economic decisions. However, it has the flexibility to be implemented less frequently at the supervisory level because controller set-point trajectories are the primary decision variables of the economic optimization problem. This allows the existing plant automation architecture to remain unaltered, and the higher frequency control calculation remains less complex and computationally inexpensive.

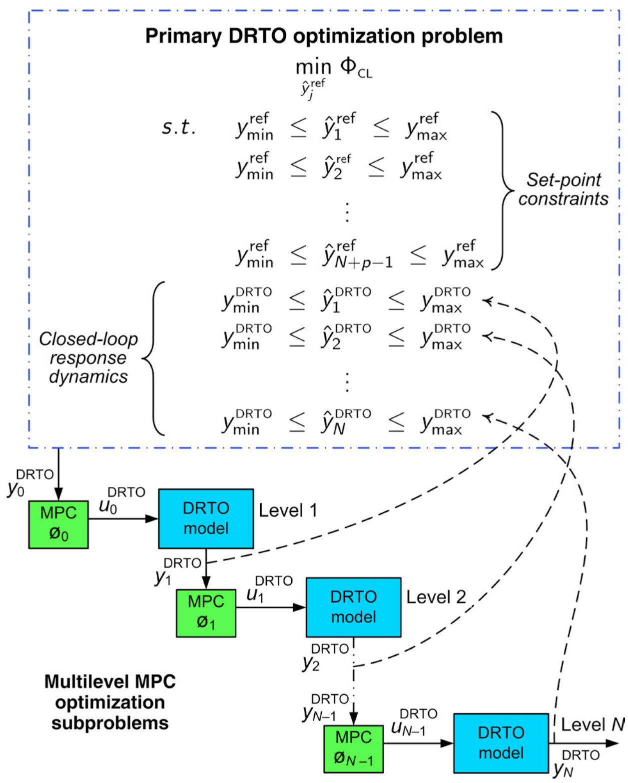

Fig. 1. Illustration of the multilevel program of the closed-loop DRTO problem with embedded MPC optimization subproblems

Our prior analysis based on a linear dynamic system demonstrated that the closed-loop DRTO strategy outperforms the traditional open-loop counterpart under control performance limitations where the controller has to be detuned (Jamaludin and Swartz, 2014, 2015b). Despite its advantages, the multilevel programming formulation significantly increases the size and solution time of the DRTO problem. In a recent study (Jamaludin and Swartz, 2015a), we proposed a closed-loop DRTO strategy in the form of a bilevel programming problem in which only a single MPC calculation is embedded over the DRTO optimization horizon as an approximation of the rigorous closed-loop response dynamics.

This paper extends the application of the closed-loop DRTO strategy to a nonlinear dynamic system formulated as a bilevel program. We consider implementation of linear MPC on a nonlinear dynamic plant model. The following sections describe the closed-loop formulation of the nonlinear DRTO problem, followed by the solution approach adopted for the resulting dynamic optimization problem. A technique to handle complementary constraints arising from reformulation of the MPC optimization subproblem to its KKT conditions, and formulation of an economic objective function, are also be presented. A comparative study of the closed-loop DRTO performance based on the nonlinear and linear embedded dynamic plant model is conducted using a polystyrene grade transition case study in the presence of plant-model mismatch and a disturbance.

2. PROBLEM FORMULATION

2.1 Dynamic Optimization

For clarity of exposition, we first describe the conversion of a nonlinear continuous dynamic model to its discrete representation, which will later be used with the embedded MPC optimization subproblem. Here we consider process dynamic behavior described by a nonlinear differential algebraic equation (DAE) system of the form:

$$\begin{aligned}
\dot{\hat{x}}^{\text{DRTO}}(t) &= \mathrm{f}^{\text{DRTO}}(\hat{x}^{\text{DRTO}}(t),\ \hat{z}^{\text{DRTO}}(t),\ \hat{u}^{\text{DRTO}}(t)) \\
0 &= \mathrm{h}^{\text{DRTO}}(\hat{x}^{\text{DRTO}}(t),\ \hat{z}^{\text{DRTO}}(t),\ \hat{u}^{\text{DRTO}}(t)) \\
0 &\leq \mathrm{g}^{\text{DRTO}}(\hat{x}^{\text{DRTO}}(t),\ \hat{z}^{\text{DRTO}}(t),\ \hat{u}^{\text{DRTO}}(t)) \\
\hat{x}^{\text{DRTO}}(0) &= x_0 \\
&\text{for } t \in [0,\ t_f]
\end{aligned}$$

where $\hat{x}^{\text{DRTO}}(t) \in \mathbb{R}^{n_x}$ = differential state vector, $\hat{x}^{\text{DRTO}}(0)$ = initial state vector, $\hat{z}^{\text{DRTO}}(t) \in \mathbb{R}^{n_z}$ = algebraic state vector, $\hat{u}^{\text{DRTO}}(t) \in \mathbb{R}^{n_u}$ = control input vector, and t_f = final time in prediction/optimization horizon. To solve the DAE system using an optimization (specifically, a nonlinear programming [NLP]) framework, the differential-algebraic equations are discretized in the time coordinate using a Backward Euler approximation.

We assume that the controller sampling interval is the same duration as the finite element interval Δt_k partitioned over the optimization horizon, and thus piecewise constant inputs (i.e. zero-order hold) can be conveniently placed at every finite element. There are also other nonlinear discretization approaches such as orthogonal collocation on finite elements (which corresponds to an implicit Runge-Kutta method), should a more precise integration procedure be desired. The resulting set of equations is posed as constraints in the optimization problem. In our case, the resulting sparse structured NLP is modeled using a specialized in-house modeling software package, a Modeling Language for Dynamic Optimization (MLDO) (Chong and Swartz, 2006), which generates AMPL code, permitting solution by means of large-scale NLP solvers.

The bilevel closed-loop DRTO formulation consists of a primary DRTO optimization problem based on the nonlinear dynamic system (1a) to predict the closed-loop response dynamics, and an inner MPC optimization subproblem based on the linear(ized) dynamic system to calculate the optimal control input trajectories (1b). The controller set-point trajectories $\hat{y}^{\text{SP}}$ are the decision variables for the outer problem, whereas the control input trajectories $\hat{u}_k^{\text{MPC}}$ are the decision variables for the inner subproblem.

Φ^{DRTO} is a purely economic objective function. $\tilde{\mathrm{f}}^{\text{DRTO}}$ represents the dynamic model utilized for DRTO prediction