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Performance Improvement of Extremum Seeking Control using Recursive Least Square Estimation with Forgetting Factor

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Abstract: The main limitation of perturbation based extremum seeking methods is the requirement of a multiple time-scale separation between the system dynamics, the perturbation frequency, and the adaptation rate so as to avoid interactions and possible instabilities. This causes the convergence to be extremely slow. In the present work, we propose a simple modification to the perturbation-based extremum seeking control method that can be used when the system cannot be accurately approximated by a Wiener-Hammerstein model for which convergence rate acceleration schemes are available. The linear filtering used in the perturbation based extremum seeking control for estimating the objective function gradient is replaced by a recursive least square with forgetting factor estimation algorithm. It is shown that this simple modification can accelerate convergence to the optimum by removing one time scale separation.

1. INTRODUCTION

After the publication of (Krstic and Wang (2000)) in which a formal proof of convergence has been established, perturbation based extremum seeking methods (Blackman (1962)) became once more an active and popular field of research in the control community. The main limitation of this class of extremum seeking methods remains the requirement of a multiple time-scale separation between the system dynamics, the perturbation frequency, and the adaptation rate so as to avoid interactions and possible instabilities (Krstic and Wang (2000)). The perturbation frequency should be slow enough to consider the system to be a static one. This, in turn, causes the convergence to be extremely slow, i.e. two orders of magnitude slower than the system dynamics. Though this is acceptable for fast systems whose time constants range in seconds (typically found in mechanical and electrical systems) (Wang et al. (2000)), it becomes unacceptable for chemical or biochemical systems where the time constants are in hours or days (Dochain et al. (2011)). This means that one would need a month to a year to complete an optimization cycle. Two concepts have been used to improve the convergence rate of extremum-seeking control using perturbations. Firstly, control algorithms can be applied to the system in order to accelerate its dynamics and increase its bandwidth (see e.g.(Krstic (2000); Chioua et al. (2007b))). Secondly, a phase compensator can be used to correct for the phase shift introduced by the system dynamics at the perturbation frequency (Aryur and Krstic (2003); Krstic (2000)). Control algorithms seek to reduce the phase shift for a wider range of frequencies but are limited by the system relative degree, the internal dynamics stability and the presence of delays. Phase compensator, on the other hand, concentrates on the given perturbation frequency and requires the knowledge of the phase shift at that frequency.

In (Atta et al. (2014)) and (Chioua et al. (2007a)), the authors follow the phase compensation idea, but instead of relying on an a priori value, the phase shift is estimated based on the available measurements. In (Atta et al. (2014)), a Kalman filter is used to estimate the phase and amplitude of the first harmonic of the system output. In (Chioua et al. (2007a)), the phase is estimated by modulating the output with the quadrature of the perturbation signal. A low frequency perturbation is added to determine the sign of the gradient which is in turn necessary for the stability of the phase adaptation loop. Applicability of the phase compensation method proposed in (Chioua et al. (2007a)) is restricted to systems that can be approximated by a Wiener-Hammerstein model.

Recently, in (Guay and Dochain (2015); Ehsan and M.Guay (2014)) the authors reformulate the extremum seeking control problem as a time-varying estimation problem.

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In the the present work, we propose a simple modification to the perturbation-based extremum seeking control method that can be used when the system cannot be accurately approximated by a Wiener-Hammerstein model. We suggest to replace the linear filtering used in the perturbation based extremum seeking control method for the gradient estimation by a recursive least square with forgetting factor estimation and show that this leads to an acceleration of convergence to the optimum by removing one time scale separation.
The paper is organized as follows: The next section introduces the traditional perturbation method for extremum seeking. Section 3 presents the modified perturbation method with recursive least squares estimation for which a convergence analysis is provided in Section 4. A simple example is presented in Section 5 and Section 6 concludes the paper.

2. EXTREMUM SEEKING USING PERTURBATIONS

The problem addressed is the steady-state optimization of a nonlinear dynamic system as stated below:

$$\begin{aligned} &\min_u J(x,u) \\ &s.t.\ \dot{x} = F(x,u) \equiv 0 \end{aligned} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $F : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a smooth function describing the dynamics and $J : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}$ the objective function.

To solve this optimization problem online, the following extremum-seeking controller is derived from the necessary conditions of optimality, under the assumption that the function J is convex.

$$\dot{u} = -k\frac{dJ}{du} = -k\left(\frac{\partial J}{\partial u} - \frac{\partial J}{\partial x}\left(\frac{\partial F}{\partial x}\right)^{-1}\frac{\partial F}{\partial u}\right) \quad (2)$$

The perturbation based methods add an excitation signal to the input in order to extract the gradient (Fig. 1). Note that the objective function is supposed to be available or directly measured ($y = J(x,u)$).

A high pass filter with cutoff frequency ω_h isolates the variations of this optimized variable from its average value. The state that represents the high pass filter is denoted by η. This signal is then modulated by the same excitation signal. A low pass filter with cutoff frequency ω_l and output ξ will filter the resulting signal in order to get the required gradient, $\xi = \frac{dJ}{du}$. Finally, an integral controller with gain k drives this estimated gradient to zero.

The method can be summarized using the following equations:

$$\dot{\hat{u}} = -k\xi, \qquad u = \hat{u} + a\sin(\omega t) \quad (3)$$

$$\dot{\xi} = -\omega_l\xi + \omega_l(y-\eta)a\sin(\omega t) \quad (4)$$

$$\dot{\eta} = -\omega_h\eta + \omega_h y \quad (5)$$

The value of the states at steady-state obtained from $\dot{x} = F(x,u) \equiv 0$ is given by $x = l(u)$. Then, the cost function at steady-state is given by $y = J(l(u),u)$.

Next the deviation variables are defined: $\tilde{u} = \hat{u} - u^*$, $\tilde{y} = y - y^*$ and $\tilde{\eta} = \eta - y^*$ where y^* is the minimum value of the cost function at steady-state y obtained for $u = u^*$. Then, the relationship between $\tilde{y}$ and $\tilde{u}$ is expressed as $\tilde{y} = J(l(u^*+\tilde{u}), u^*+\tilde{u}) - J(l(u^*),u^*) \equiv \nu(\tilde{u})$.

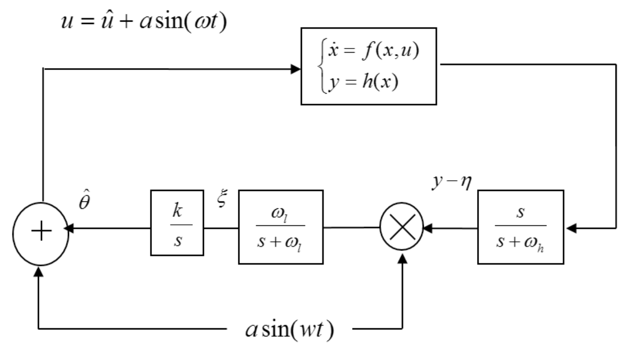

Fig. 1. Extremum seeking control via perturbation method inspired from (Krstic and Wang 2000).

Assuming that x is at steady state, the averaged system for the three remaining variables (u, ξ, and η) is obtained by taking the average of the right hand side over $[0, \frac{2\pi}{\omega}]$. The averaged states are denoted by the superscript $(\cdot)^a$. The averaged system reads (Khalil (2002)):

$$\frac{d}{dt}\begin{bmatrix}\tilde{u}^a \\ \xi^a \\ \tilde{\eta}^a\end{bmatrix} = \begin{bmatrix} -k\xi^a \\ -\omega_l\xi^a + \frac{\omega_l\omega}{2\pi}a\int_0^{\frac{2\pi}{\omega}}\nu(\tilde{u})\sin(\omega t)dt \\ -\omega_h\tilde{\eta}^a + \frac{\omega_h\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}}\nu(\tilde{u})dt \end{bmatrix} \quad (6)$$

where $\tilde{u} = \tilde{u}^a + a\sin(\omega t)$.

Convergence is established in (Krstic and Wang (2000)) through the following steps:

- The exponential stability of the equilibrium point of the above averaged system ($\tilde{u}^a$, ξ^a, and $\tilde{\eta}^a$) is first proved.
- From there on, the exponentially stability of (u, ξ, and η) (non-averaged) is established using the averaging theorem (Khalil (2002)).
- This non-averaged system (u, ξ, and η) acts as the "slow" manifold, while the original system $\dot{x} = F(x,u)$ acts as the boundary layer system which is assumed to be exponentially stable. Then, singular perturbation ideas are applied to show that their interconnection is also exponentially stable (Khalil (2002)).

The key assumption is that the system is at quasi-steady state, i.e., to a second order approximation,

$$\nu(\tilde{u}) = \frac{1}{2}\nu''((\tilde{u}^a)^2 + \frac{a^2}{2} + 2\tilde{u}^a a\sin(\omega t) - \frac{a^2}{2}\cos(2\omega t)) \quad (7)$$

But due to the dynamics of the system, the sinusoids of frequencies ω and 2ω will have phase shifts which will affect convergence. So, instead of using $\nu(\tilde{u})$ the following dynamic operator $P(\tilde{u})$ is used:

$$\begin{aligned} P(\tilde{u}) = &\frac{1}{2}G_0((\tilde{u}^a)^2 + \frac{a^2}{2}) + 2G_\omega\tilde{u}^a a\sin(\omega t - \varphi_\omega) \\ &- G_{2\omega}\frac{a^2}{2}\cos(2\omega t - \varphi_{2\omega})) \end{aligned} \quad (8)$$