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Diagnosability and detectability of multiple faults in nonlinear models

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ABSTRACT

This paper presents a novel method for assessing multiple fault diagnosability and detectability of nonlinear parametrized dynamical models. This method is based on computer algebra algorithms which return precomputed values of algebraic expressions characterizing the presence of some constant multiple fault(s). Estimations of these expressions, obtained from input and output measurements, permit then the detection and the isolation of multiple faults acting on the system. The application of this method on a coupled water-tank model attests the relevance of the suggested approach.

1. Introduction

The problem of fault-diagnosis has received an increasing attention during the recent years in order to increase security of systems, to monitor their performance or to endow them with self diagnostic capabilities. To answer such technological requirements, this problem needs to be taken into account in the system design stage from an a priori diagnosability study on a model. In studying anticipated fault situations from different symptoms of the system, faults or multiple faults can be known as discriminable according to the available sensors in a system. Some procedures for detecting and isolating them may, then, be put in place in the design stage. By this way, diagnosability can permit to anticipate component failures.

In this paper, we assume to be in the model-based framework and, more precisely, that available input and output signals and a nonlinear parametrized dynamical model permit to reach the output trajectories of the system. The problem consists in this framework to evaluate diagnostic performance given a model only. By (multiple) fault, we mean any change(s) of parameter value(s) implying unwanted changes in the behavior of one or more component(s) of the system. The *fault diagnosis* study consists in two subtasks [1]. The first one concerns the *fault detection* (FD) of the malfunction, the second one the *fault isolation* (FI) of the faulty component (that is the determination of its location). The fault diagnosis is done from the comparison between predictions of the model and behaviors of the system. Several methods are proposed in the literature as nonlinear observers [2] or methods based on testable subsets of equations [3]. The issue of subsets generation has been studied by many authors. They can be based on Minimal Structurally Overdetermined sets (MSO) [4,5], on possible conflicts [6] or on Analytical Redundancy Relations (ARRs) [7]. The latter are relations linking inputs, outputs, their derivatives, the parameters of the model and the faults (See [8–12] for single faults and [13–15] for multiple faults). Some of these ARRs are obtained using computer algebra tools such as the Rosenfeld–Groebner algorithm which permits to eliminate the unknown variables of the model (See [16–18]).

With respect to a specific elimination order, this algorithm returns particular differential polynomials classically called *input–output polynomials*. Some recent works have already used these particular polynomials in diagnosis assuming that the model is identifiable with respect to the faults (See [11]). Indeed, identifiability insures that the fault values can be uniquely inferred from input–output measurements. In the case of single faults, authors in [11] prove that if the model is identifiable with respect to the faults then all the faults are discriminable; in other words, the model is diagnosable. Furthermore, they prove that the residuals associated to each ARR permit to detect each identifiable fault in adopting a discriminable behavior. In [19], assuming that the faults act only additively on parameters, detectability is obtained directly from the ARRs. In this last paper, interval analysis is used to estimate the simple faults.

We propose a new approach to exploit such ARRs to discriminate, detect and isolate (multiple) fault(s) in models not necessary identifiable. Starting from the model, the three steps of our method

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described hereafter can be completely automatized; our contributions consist in the steps (2) and (3).

1. The first one is the computation of ARRs by applying the Rosenfeld–Groebner algorithm to the model.
2. The second step consists in using Groebner basis computations in order to obtain an algebraic application called *algebraic signature*. Each of its components depends only on the parameters and on the coefficients of the ARRs and can be numerically estimated from the input and output measurements of the system. By construction, each component of the algebraic signature vanishes when at least one specific (multiple) fault occurs.
3. For each possible (multiple) fault, the third step consists in using semialgebraic set tools to certify that some components of the signature vanish or never vanish. These expected values can be summarized in a precomputed table.

This table constitutes the input of the numerical treatment which, from the system measurements, returns estimations of the algebraic signatures. Their comparison with their nominal values permits to detect and isolate (multiple) fault(s).

Our method is not based on the direct use of the functional ARRs (see [8–15]) but only on algebraic relations deduced from ARRs coefficients. This change of point of view is a first advantage since it permits the complete processing of the algebraic relations to study diagnosability. Owing to the use of semialgebraic set tools in this processing, constraints on parameters and multiple faults, such as inequalities satisfied by parameters or constraints deduced from initial conditions, can be taken into account through automatic procedures. These constraints can play a fundamental role in the FDI analysis of a model as it is shown in this paper. The second advantage of the present method is not to require any strong assumption on the model for determining some possible acting multiple fault(s) as (i) its identifiability, (ii) the value of some model parameters in some particular cases, (iii) the additive action of multiple faults on parameters. Finally, from an a priori study on the model, a numerical method based only on estimation of algebraic expressions, and consequently fast, is proposed to do FDI.

The paper is organized as follows. In Section 2, we present the framework of our method. In this section, we precise the assumptions on the dynamical models and on the constraints that must be verified by parameters and faults. Section 3 is devoted to our method consisting in studying the diagnosability of a model, that is the way to compute an algebraic signature and to tabulate its expected values in function of the multiple faults. In Section 4, our method is applied to an example of two coupled water-tanks. Section 5 concludes the paper. In this paper, the symbolic computations had been realized with Maple 18 and the numerical part with Scilab.

2. Dynamical models and algebra concepts

We consider nonlinear parametrized models controlled or uncontrolled of the following form:

$$\Gamma_f \begin{cases} \dot{x}(t,p,f) = g(x(t,p,f),u(t),p,f), \\ y(t,p,f) = h(x(t,p,f),u(t),p,f), \\ t_0 \le t \le T \end{cases} \tag{1}$$

where:

- the vector of real parameters $p = (p_1, \ldots, p_m)$ belongs to $P \subseteq \mathbb{R}^m$ where P is an *a priori* known set of admissible parameters,
- $f = (f_1, \ldots, f_e)$ is a constant fault vector which belongs to a subset F of $\mathbb{R}^e$. It is equal to 0 when there is no fault. The set F describes the set of admissible values of the fault vectors f,
- $x(t,p,f) \in \mathbb{R}^n$ denotes the state variables and $y(t,p,f) \in \mathbb{R}^s$ the outputs,
- g and h are real vectors of rational analytical functions in x, p and f[1].
- $u(t) \in \mathbb{R}^r$ is the control vector equal to 0 in the case of uncontrolled models.

Remark 1. In most practical cases, the faults f_i belong to connected sets of $\mathbb{R}$, and F is the Cartesian product of these sets. The present work takes place in a more general framework by introducing semialgebraic sets defined hereafter.

From now on, we suppose that constraints on $p \in P$ and $f \in F$, and eventual constraints linking fault and parameter components, can be formulated by the mean of algebraic equations and/or inequalities. This consideration leads naturally to consider semialgebraic sets for which computer algebra tools are developed (See [21–23] for example):

Definition 1. (See [24]) Let $\mathbb{R}[X_1, \ldots, X_n]$ the set of polynomials with real coefficients and where $X_1, \ldots, X_n$ are n indeterminates.

A set of real solutions of a finite set of multi-variable polynomial equations (of the form $P = 0$) and/or polynomial inequalities (of the form $Q \ge 0$) of $\mathbb{R}[X_1, \ldots, X_n]$ is called a semialgebraic set.

Let C_{pf} be the set of all algebraic equations and inequalities verified by the components of the parameter and fault vectors of the model and $\mathcal{C}_{p,f}$ be the semialgebraic set defined by C_{pf}. In order to take into account initial conditions, the algebraic relations induced by these conditions can be added to the set C_{pf}.

Example 1. For illustrating the theoretical part, the example of a mass $m = 1$ attached to an elastic spring of force k is considered. Denote u an external force of the system not identically equal to zero and $d \ge 1$ a constant. The move of the mass is described by the following equation

$$\ddot{x} + kx - du = 0 \tag{2}$$

which can be rewritten as model (1):

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -kx_1 + du. \end{cases} \tag{3}$$

Assume that two faults $f_1 \in [0,2)$ and $f_2 \in [0,2)$ impact respectively the spring k and the parameter d such that the model takes the form:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -k(f_1 - 1)^2 x_1 + (d + f_2)u. \end{cases} \tag{4}$$

The parameters, k and d and the faults, f_1 and f_2 verify the following constraints:

$$\begin{cases} 0 < k < 4 \\ 1 \le d \\ 0 \le f_1 < 2, \\ 0 \le f_2 < 2. \end{cases} \tag{5}$$

These algebraic constraints can be viewed as a set of polynomial equations and inequalities whose indeterminates are k, d, f_1 f_2. Those algebraic inequalities define C_{pf} and the corresponding set of admissible values of the parameters and faults is $\mathcal{C}_{p,f}$.

Let N be a subset of $\{1, \ldots, e\}$ and f_N the *multiple fault vector* whose components f_i are not equal to 0 if $i \in N$ and equal to 0 otherwise. Naturally, f_N belongs to $F_N = \{f \in F | f_i \neq 0$ if $i \in N$ and $f_i =$

[1] The rational assumption is not restrictive since lots of models can be reduced to a rational model by variable change (See [20]). The analytical assumption is required to obtain ARRs by the first step of our method.