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Output feedback receding horizon regulation via moving horizon estimation and model predictive control

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ABSTRACT

This manuscript develops an algorithm that fuses Carleman moving horizon estimation (CMHE) and Carleman model predictive control (CMPC) together, to design an output feedback receding horizon controller. CMHE identifies the system states as the initial condition for CMPC to make optimal control decisions. The control decisions made by CMPC update the dynamic models used in CMHE to make more precise estimations. Modeling the nonlinear system with Carleman approximation, we estimate the system evolution for both CMHE and CMPC analytically. The Gradient vectors and Hessian matrices are then provided to facilitate the optimizations. To further reduce real-time computation, we adapt the advanced-step NMHE and advanced-step NMPC concepts to our CMHE/CMPC pair to develop an asCMHE/asCMPC pair. It pre-estimates the states and pre-designs the manipulated input sequence one step in advance with analytical models, and then it updates the estimation and control decisions almost in the real-time with pre-calculated analytical sensitivities. A nonlinear CSTR is studied as the illustration example. With CMHE/CMPC pair, the computational time is decreased to one order of magnitude smaller than standard nonlinear MHE and nonlinear MPC. With asCMHE/asCMPC pair, the real-time estimation and control decisions takes a negligible amount of wall-clock time.

1. Introduction

Model predictive control (MPC) is an optimization-based strategy in control engineering. It has been attracting attention for its readiness in dealing with multi-input-multi-output systems, in handling various bounds, in rejecting disturbances and in tolerating model-mismatches. Despite the many advantages it has, the application in industrial environment has been limited. One of the major barriers is the heavy computational burden in solving the associated dynamic optimization problem in real-time.

Researchers have developed many approaches to achieve computational acceleration. The advanced-step NMPC (asNMPC) algorithm, published by Biegler and coworkers in [1–4], has focused on solving complex optimization problems off-line then performing an update with linear approximation of nonlinear sensitivity in real-time. Multi-parametric MPC developed by Pistikopoulos and coworkers in [5,6] accelerates computation via querying response hypersurfaces. Fast approaches to solve MPC have also been reported in applications, such as extended differential flatness approach in control and estimation of lithium-ion batteries. [7–9]

Most MPC designs assume that all system states are measurable and immediately available at the beginning of each sampling time. This assumption is rarely true in industrial practice. Estimators are required to obtain the information of the system state. In addition, estimators help reducing the effect of model mismatches and unknown disturbances [10]. It is reasonable to integrate the design of controller with an estimator to account for the lack of state information. Huang et al. presented an Extended Kalman Filter (EKF) and NMPC scheme in [11,12]. Haseltine et al. reported a critical evaluation of EKF vs. MHE in [13]. MHE is an optimization-based estimation method. It uses limited information regarding the input, output and plant model to discover system states. The basic design of MHE is similar to that of MPC. It also handles constraints and bounds in a straightforward manner, but instead of predicting into the future, MHE uses a sliding window of outputs into the past. Despite the ongoing debate between EKF and MHE [14], for the purpose of developing an estimation and control pair, we pick MHE as the ideal technology to combine with MPC, since they share the same state-space model.

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A basic MHE and MPC pair has the following interconnection. The state estimation identified by MHE serves as the initial condition of each receding time window for MPC to make optimal control decisions. As the process recedes, these control decisions are continuously updated in the process model used by MHE to make MHE more precise. This way, the interconnection between MHE and MPC is completed. MHE and MPC are designed separately, aligning with the Separation Principle, but they collaborate closely with each other to form the MHE/MPC pair. Expectedly, MHE has computational delay issues for the same reasons as MPC. That is a significant obstacle for the application of MHE/MPC pair.

Motivated by the above issue, we propose bilinear Carleman approximation to formulate a new scheme of the MHE and MPC pair, CMHE and CMPC. Carleman approximation is also known as Carleman linearization. [15–17]; the theoretical basis is to express a finite dimensional nonlinear system with an infinite dimensional linear one. It carries nonlinear dynamic information of the original system, which reduces approximation errors compared with classic linearization methods. Our previous work has reported the structure of CMPC [18], explored its formulation [19,20] and sensitivity analysis [21]. We also investigated the structure of CMHE [22], its application [23], and stability analysis [24]. In this manuscript, we improve our previous work by proposing a CMHE/CMPC pair, fusing these two algorithms together. We also extend the fused algorithm with advanced-step NMHE (asNMHE)[1] and NMPC (asNMPC) [2,3] concept to develop an asCMHE/asCMPC pair. The developed pair further reduces real-time computational efforts to a negligible amount, with significant impact on its applicability to fast processes.

This manuscript is organized as follows: Section 2 introduces the preliminary information of our proposed algorithms. Section 3 describes the formulation of the CMHE/CMPC pair. It also presents the derivations of the Gradient vector and the Hessian matrix. Section 4 presents the strategy and detailed algorithm of the asCMHE/asCMPC pair. Section 5 studies a nonlinear CSTR system as simulation examples of the applications of the CMHE/CMPC pair, as well as the asCMHE/asCMPC pair. Section 6 concludes the manuscript with a summary and future directions.

2. Preliminary information

2.1. Nonlinear system under investigation

Consider a nonlinear system with noise,

$$\dot{x} = f(x) + g(x)u + \gamma(x)w \tag{1}$$

$$y = h(x) + \nu \tag{2}$$

$$x(t_0) = x_0 \tag{3}$$

where $x \in R^{n_x}$ is the state vector; $u \in R^m$ is the vector of manipulated variables, $u = [u_1 u_2 \ldots u_m]$; $w \in R^n$ is the vector of unknown process noise, $w = [w_1 w_2 \ldots w_n]$; $y \in R^p$ is the output vector; $\nu \in R^p$ is the vector of unknown sensor noise; x_0 is the initial condition at the beginning time t_0. $f(x)$ is a nonlinear vector function; $g(x)$ and $\gamma(x)$ are nonlinear matrix functions. In this manuscript, we assume $f(x)$, $g(x)$, $\gamma(x)$ and $h(x)$ are known and locally analytic.

For simplicity of the presented formulations later, we expand matrix functions $g(x)$ and $\gamma(x)$ as summations of vector functions, $g_j(x)$, $j = 1, \ldots, m$ and $\gamma_l(x)$, $l = 1, \ldots, n$. Eq. (1) is re-expressed as:

$$\dot{x} = f(x) + \sum_{j=1}^{m} g_j(x)u_j + \sum_{l=1}^{n} \gamma_l(x)w_l \tag{4}$$

$$y = h(x) + \nu \tag{5}$$

Remark 1. In this manuscript, we assume both the process noise w and the sensor noise ν are bounded in compact sets. There are no limitations on their distributions within the bounds.

2.2. Mathematics preliminary: Carleman approximation

The mathematical foundation of our control approach is Carleman approximation (the Kronecker product rule presented in Appendix A for completeness). Carleman approximation is a two tier approach: first, we choose a desired steady state point to perform Taylor expansion to the nonlinear system under investigation. This step naturally expresses the original state vector as deviations from the desired steady state. These deviation terms are in a polynomial form and contain higher orders. Second, we expand both the transfered state vector and the coefficient matrices based on the Kronecker product rule. This results in a bilinear system that has a larger dimension.

To implement Carleman approximation, the states of the system x are extended to

$$x_{\otimes} = [x^T x^{[2]^T} \ldots x^{[p]^T}]^T, \tag{6}$$

where $x^{[p]} = x \otimes x^{[p-1]}$ denotes the pth order Kronecker product of x.

For simplicity of presentation and without loss of generality, we assume the nominal operating point is at the origin $x = 0$. Nonlinear vector functions $f(x)$, $g_j(x)$, $\gamma_l(x)$ and $h(x)$ are expanded by Maclaurin series in the following form:

$$f(x) = f(0) + \sum_{k=1}^{\infty} \frac{1}{k!} \partial f_{[k]} \Big|_{x=0} x^{[k]} \tag{7}$$

$$g_j(x) = g_j(0) + \sum_{k=1}^{\infty} \frac{1}{k!} \partial g_{j[k]} \Big|_{x=0} x^{[k]} \tag{8}$$

$$\gamma_l(x) = \gamma_l(0) + \sum_{k=1}^{\infty} \frac{1}{k!} \partial \gamma_{l[k]} \Big|_{x=0} x^{[k]} \tag{9}$$

$$h(x) = h(0) + \sum_{k=1}^{\infty} \frac{1}{k!} \partial h_{[k]} \Big|_{x=0} x^{[k]} \tag{10}$$

As mentioned earlier, we assume $f(x)$, $g_j(x)$, $\gamma_l(x)$ and $h(x)$ are analytic functions (i.e., Taylor expansion is locally convergent), so nonlinear dynamic systems of Eq. (4)(5) can be approximated by a polynomial form with arbitrarily chosen accuracy:

$$\dot{x} \cong \sum_{k=0}^{p} A_k x^{[k]} + \sum_{j=1}^{m}\sum_{k=0}^{p} B_{jk} x^{[k]} u_j + \sum_{l=1}^{n}\sum_{k=0}^{p} D_{lk} x^{[k]} w_l \tag{11}$$

$$y \cong \sum_{k=1}^{p} C_k x^{[k]} + \nu \tag{12}$$

$A_k = \frac{1}{k!}\partial f_{[k]}|_{x=0}$; $B_{jk} = \frac{1}{k!}\partial g_{j[k]}|_{x=0}$; $D_{lk} = \frac{1}{k!}\partial \gamma_{l[k]}|_{x=0}$; $C_k = \frac{1}{k!}\partial h_{[k]}|_{x=0}$. $A_0 = f(0)$; $B_{j0} = g_j(0)$; $D_{l0} = \gamma_l(0)$.

The polynomial order p is assumed to be high enough to reduce truncation errors to below a chosen threshold [17].

A notable detail: $\partial f_{[k]}$, $\partial g_{j[k]}$, $\partial \gamma_{l[k]}$ and $\partial h_{[k]}$ are derivatives based on the Kronecker product rule.

$$\partial f_{[k]} = \frac{\partial}{\partial x} \otimes \partial f_{[k-1]}, \quad \partial g_{j[k]} = \frac{\partial}{\partial x} \otimes \partial g_{j[k-1]}, \tag{13}$$

$$\partial \gamma_{l[k]} = \frac{\partial}{\partial x} \otimes \partial \gamma_{l[k-1]}, \quad \partial h_{[k]} = \frac{\partial}{\partial x} \otimes \partial h_{[k-1]} \tag{14}$$