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Economic model predictive control based on a periodicity constraint

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ABSTRACT

This paper addresses a novel economic model predictive control (MPC) formulation based on a periodicity constraint to achieve an optimal periodic operation for discrete-time linear systems. The proposed control strategy does not rely on forcing the terminal state by means of a terminal equality constraint and hence it does not require a priori knowledge of a periodic steady trajectory. Instead, at each sampling time step the economic cost function is optimized based on a periodicity constraint over all the periodic trajectories that include the current state. The recursive feasibility and the closed-loop convergence to a periodic steady trajectory are discussed. Moreover, an optimality certificate of this steady trajectory is provided based on the Karush–Kuhn–Tucker (KKT) optimality conditions. Finally, an application to a well-known water distribution network benchmark is presented to demonstrate the proposed economic MPC in which the closed-loop simulation results obtained with a linear model and a virtual–reality simulator are both provided.

1. Introduction

Economic model predictive control (MPC) has attracted an increasing attention during the past decade [1–3]. Unlike the conventional MPC formulations [4,5], the main control objective of economic MPC is to optimize an economic performance index without regulating the system to a given steady state or trajectory. Economic cost functions are not necessarily quadratic or positive definite with respect to the given trajectories or references as tracking MPC. Economic MPC has been applied to a variety of industrial applications, such as water distribution networks [6–8], wastewater treatment processes [9], smart grids [10,11] and chemical processes [12,13].

Recently, the closed-loop stability of economic MPC has been widely investigated. Unlike the conventional MPC, the cost function may not be regarded as a quadratic function with respect to a given reference. Hence, the standard stability analysis cannot be directly applied. In [3,14,15], stability analysis of economic MPC has been established under the strong duality and the dissipativity assumptions. Terminal cost and constraint around the optimal steady state are used. In [16], a review is presented for discussing the role of constraints in economic MPC, where the convergence of economic MPC can be enforced by adding terminal constraints. Besides, economic MPC without terminal constraints is studied in [17,18]. Based on the turnpike and controllability properties, the closed-loop convergence is proved. In [19], economic MPC with extended prediction horizon is designed based on an auxiliary controller. An additional term with the auxiliary control law is included in the cost function in order to guarantee the closed-loop convergence.

From the application point of view, periodic system behavior appears in some specific systems, such as water distribution networks (WDNs) [20,21] and electrical networks [10]. One specific example stems from the periodic behavior of customer demands in water distribution networks. A WDN generally consists of a large number of hydraulic elements, such as storage tanks, pressurized pipelines, pumping stations (including several parallel pumps) and valves. Economic MPC is suitable for optimizing the economic performance of operations in WDNs, as shown in [6,7,22], but these methods do not take specific advantage of the periodic nature of the consumer demands and energy costs. Taking into account the daily water demand patterns and periodic electricity prices, periodic operations can also be considered in the economic MPC design. A study of periodic economic MPC for the management of a WDN has been reported in [23], where the nonlinear model of a WDN is used with differential and algebraic equations. The closed-loop

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simulations show that optimal periodic operation achieved by an economic MPC controller is also appearing in the operational management of WDNs.

In the literature, the Lyapunov stability of economic MPC with periodic operation is discussed for nonlinear processes in [24]. In [20], a single-layer economic MPC is proposed for periodic operation. An online planner is included in the corresponding optimization problem in order to find the open-loop optimal periodic steady states that are subsequently set as the terminal states for economic MPC. This single-layer strategy for periodic operation is used in [25], where economic MPC without terminal constraints is designed for optimal periodic behavior under the dissipativity and controllability conditions. The closed-loop trajectory converges to the optimal periodic orbit.

The main contribution of this paper is to propose a novel economic MPC based on a periodicity constraint for discrete-time linear systems. We formulate an economic MPC optimization problem without setting a terminal state. Hence, it does not need to know a periodic steady trajectory as a priori knowledge. Therefore, the economic cost function is optimized with a periodicity constraint considering all the periodic trajectories including the current state along the prediction horizon. Different from the conventional MPC optimization formulation, the current state is set as a shifted position and not necessarily being the first prediction state. The recursive feasibility and the closed-loop convergence to a periodic steady trajectory are discussed. In order to investigate the optimality of this periodic steady trajectory, an optimal periodic steady trajectory can be obtained by solving an open-loop finite-horizon optimization problem. And an optimality certificate is provided based on the Karush–Kuhn–Tucker (KKT) optimality conditions [26,Section 5.5.3].

Furthermore, an application to the Richmond WDN is presented with the proposed economic MPC controller. Two closed-loop simulation results are provided with linear control-oriented model and with a virtual–reality hydraulic simulator, EPANET. In the case of the linear control-oriented model, no modeling errors and system uncertainties are taken into account. The expected result is that the closed-loop trajectory can converge to the planner trajectory as well as the MPC cost and the closed-loop operational cost. In the case of EPANET, because of the mismatches between the prediction model in economic MPC controller and the simulation model in EPANET, the closed-loop trajectory could not be perfectly periodic but the cost converges to the optimal one provided by the planner.

The remainder of this paper is organized as follows. In Section 2, the problem statement of economic MPC is expressed. The economic MPC controller is proposed in Section 3 and the recursive feasibility and convergence analysis of the proposed economic MPC are discussed in Section 4. The simulation results of applying the proposed economic MPC into the Richmond WDN are shown in Section 5. Finally, some conclusions are highlighted in Section 6.

2. Problem statement

Consider the class of discrete linear time-invariant systems

$$x_{k+1} = Ax_k + Bu_k, \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ and $u \in \mathbb{R}^{n_u}$ denote the system state vector and the control input vector, respectively. Moreover, $A \in \mathbb{R}^{n_x \times n_x}$ and $B \in \mathbb{R}\mathbb{R}^{n_u \times n_u}$ are system matrices.

For the system (1), system states and control inputs are limited by the following constraints:

$$x_k \in X, u_k \in U, \quad \forall k \in \mathbb{N}, \tag{2}$$

where X and U are strictly convex sets of states and inputs.

The economic performance of the system (1) is measured by a time-varying economic cost function

$$\ell_k(x_k, u_k, p_i), \quad i = \mathrm{mod}(k, T) \tag{3}$$

where $T \in \mathbb{Z}_+$ is a period index and p_i is a time-varying exogenous signal usually indicating the unit prices, which is stored in a known sequence p as

$$p = \left\{p_i\right\}, \quad i = 1, \ldots, T,$$

and exhibiting a periodic behavior is implemented using the modulo operator mod(k, T). It is worth mentioning that $\ell_k(x_k,\ u_k,\ p_i)$ is not necessarily a quadratic function that depends on a sequence of references for tracking. The main control objective is to minimize the closed-loop economic cost measured by $\ell_k(x_k,\ u_k,\ p_i)$ that is a strictly convex function, $\forall k \in \mathbb{N}$ and the periodicity of this economic stage cost function is given by $\ell_k(x_k, u_k, p_i) = \ell_{k+T}(x_{k+T}, u_{k+T}, p_i)$ with $i = \mathrm{mod}(k, T)$.

In this paper, we propose an economic MPC formulation with guaranteeing the closed-loop system convergence to a periodic steady trajectory that minimizes the economic cost while satisfying all the constraints. A procedure to certify that the reached trajectory is optimal with respect to the economic cost provided. In addition, the proposed controller does not lose feasibility even in the presence of sudden changes in the economic cost.

3. Economic model predictive control based on a periodicity constraint

In principle, MPC controllers are based on solving a finite horizon optimization problem. If a steady-state trajectory is known, a terminal constraint is included forcing the predictions to reach this steady trajectory at the end of the MPC prediction horizon. While several controllers proposed in the literature are based on a standard terminal region/constraint approach, in this paper a periodic steady-state trajectory is assumed to be unknown in the economic MPC design. We propose a different approach in which the MPC controller seeks to minimize the economic cost function over a single period that includes the current state. Besides, an open-loop finite-time optimization problem is also proposed to find an optimal periodic steady trajectory that will be used for the analysis of the closed-loop convergence.

The proposed controller guarantees recursive feasibility and hence the closed-loop convergence even in the presence of sudden changes in the economic cost function, because the constraints of the optimization problem are independent of this cost function. Note that standard approaches that depend on terminal constraints often lead to optimization problems that have to be modified if the economic cost function changes, which in general lead to a more complex control scheme and even to a possible loss of feasibility issues [1,25].

3.1. *Optimal periodic steady trajectory*

To find the optimal periodic steady trajectory, an open-loop finite-horizon optimization problem, the so-called *planner*, is presented. Because of the periodic nature discussed above, it can be proved that the infinite horizon problem is equivalent to the following finite horizon optimization problem in which a single period is