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IFAC-PapersOnLine 49-5 (2016) 013–018

Optimal-Performance Takagi-Sugeno Models via the LMI Null Space ⋆

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Abstract: The problem of achieving optimal performance for nonlinear systems by constructing the most adequate exact Takagi-Sugeno model of the plant and considering its relationship with the linear matrix inequalities it gives raise to, is considered in this report. In contrast with recent approaches on the subject, the performance goal can be chosen from a wide variety of definitions while constraints are no longer required to be state-dependent. As before, this approach is based on coordinate transformations that isolate the effects of the system nonlinearities and allow optimisation with respect to some performance level, by keeping some norm close enough to the linearised performance. It is shown that the proposed methodology outperforms both ordinary "blind" TS modeling as well as former similar approaches.

Keywords: Optimal, convex, modelling, performance, linearisation.

1. INTRODUCTION

Takagi-Sugeno (TS) models appeared as a way to incorporate mathematical knowledge about nonlinear plants in the construction of a rule-based fuzzy approximation. Quadratic Lyapunov-based methodologies for analysis and design of TS models were developed (Wang et al., 1996): their focus was put on exploiting the convex structure of TS models as to obtain conditions in the form of linear matrix inequalities (LMIs), which could be efficiently solved by convex optimization techniques (Boyd et al., 1994; Tanaka and Wang, 2001). No attention was paid to the actual modelling since it was assumed a TS model was readily available: whether this came from a nonlinear system or a parameter-dependent structure, was irrelevant: functions of the nonlinearities/parameters were assumed to lie in a simplex where the convex sum property held.

In the seminal work of (Taniguchi et al., 2001), a modelling technique called *sector nonlinearity* was presented: in contrast with former approaches, this methodology allows obtaining exact convex representations of nonlinear systems within a compact set of the state space; results thus obtained were directly valid for the original nonlinear setup without further adjustment. Therefore, during the next years, mainstream research abandoned earlier model-free ideas, to concentrate on model-based methodologies, where exact TS models and LMI conditions were seen as vehicles to analyse and control general nonlinear systems (Sala et al., 2005; Guerra et al., 2015a).

Nevertheless, sector nonlinearity was not without disadvantages. Modelling was identified as one of the main reasons for conservativeness in the TS/LMI framework since a family of systems can be represented by the same TS model and different TS representations can be found for a given nonlinear expression; conservatism issues in fuzzy control are discussed in (Sala, 2009). This led to the study of wider classes of convex structures such as descriptors (Guerra et al., 2015b) and polynomial models (Sala and Ariño, 2009). Moreover, since the complexity of the TS model critically depends on the number of nonlinearities in the nonlinear system, a number of methods were developed to reduce the number of rules: empirical (Setnes et al., 1998), depending on the number of inputs (Gegov, 2007), minimising the H_2 norm between a possibly complex expression of the gain-scheduling parameters and a linear-fractional one (Petersson and Löfberg, 2009), based on higher-order singular value decomposition (SVD) in order to approximate the system with another one in tensor-product form (Nagy et al., 2009), and based on the functional principal component analysis (Escaño and Bordons, 2014), among others. The former are shape-independent approaches; when the shape of the convex functions capturing the system nonlinearities is taken into account, some relaxations can also be achieved (Bernal et al., 2009; Kruszewski et al., 2009). These issues are, however, out of the scope of this contribution, since some of them are *a posteriori* methodologies (once a first model has been obtained) and others deal with *approximations* at a finite set of points rather than *exact* models.

This work inverses the usual approach on the TS/LMI framework: instead of writing LMIs for a given TS model, it assumes that LMIs for a given (linearisation-based)

⋆ The authors are grateful to project DPI2011-27845-C02-01 from Spanish Government, Grant PROMETEOII/2013/004 from Generalitat Valenciana, the Project SEP-CONACYT CB-168406, and the scholarship program Santiago Grisolía GRISOLIA/2014/006.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.07.082

optimisation problem are already available and sets out for a quest to determine the optimal TS model that keeps the proved performance as close as possible to the linearised one (which, as proven later, is the ideal). A first answer to this problem has been offered in former works by the authors, based on subspaces of the state space in which performance is more sensible to modelling errors, see (Robles et al., 2015, 2016).

This work relaxes the condition of state-dependency of former approaches, thus allowing multiple-LMI setups which might be associated with a wider variety of performance measures. The main idea here is based on a Frobenius-norm bound on the "perturbation" that sector-nonlinearity models produce in the LMI matrices.

This paper is organized as follows: section 2 defines the class of nonlinear systems under consideration and the problem of linearised performance optimisation, establishing their relationship with TS modelling and performance optimisation based on such models; section 3 develops the main results in this report, this is to say, a more direct approach where several constraints involving the model can be exploited towards an optimisation objective to keep results as close as possible to the linearised case; the methodology is illustrated in section 4 via suitable examples; in section 5, some conclusions are drawn.

2. PRELIMINARIES AND PROBLEM STATEMENT

Nonlinear affine-in-control dynamic systems will be considered in this report, i.e.

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t) \tag{1}$$

with $x(t) \in \mathbb{R}^n$ being the state, $u(t) \in \mathbb{R}^m$ being a control input, $f : \mathbb{R}^n \mapsto \mathbb{R}^n$, having continuous second-order derivatives and $f(0) = 0$. Also, consider the linearised model of (1):

$$\dot{x} = Ax + Bu,\ A := \left.\frac{\partial f(x)}{\partial x}\right|_{x=0},\ B := \left.\frac{\partial g(x)}{\partial x}\right|_{x=0} \tag{2}$$

2.1 Linearised performance optimisation

Assumption 1. The pursued control objective is the optimisation of a performance measure γ subject to LMI constraints:

$$\begin{array}{ll} \text{minimise} & \gamma, \\ \text{subject to} & \gamma > 0,\ \zeta^T LMI(L, D, \gamma)\zeta \geq 0,\ \forall \zeta \neq 0 \end{array} \tag{3}$$

where $\zeta \in \mathbb{R}^q$, D denotes the decision variables (usually matrix variables associated to the Lyapunov function, controller gains, etc.) and L stands for constant model matrices associated to the system under consideration; for instance, L could be defined as $L := \{A, B\}$ given by the linearised system in (2) [1]. Expression $LMI(\cdot,\cdot,\cdot)$ will be assumed to be (separately) linear in its arguments. Suitable convex optimisation [2] software will be employed to find the optimal γ and D.

[1] Other problems would include additional model matrices in L, for example, matrices relating the system with output-feedback settings, disturbance rejection, exact convex representations of (1), etc. Details omitted for brevity.

[2] Although the second constraint in problem (3) is named LMI, any tractable matrix inequality constraint, such as GEVP problems, can be considered in the referred expression.

Proposition 1. The optimal performance measure γ^{opt} for the linearised model (2) is obtained when there exists D^{opt} such that conditions (3), are

$$\zeta^T LMI(L, D^{opt}, \gamma^{opt})\zeta = 0 \quad \forall \zeta \neq 0, \zeta \in \mathcal{C} \tag{4}$$

$$\zeta^T LMI(L, D^{opt}, \gamma^{opt})\zeta > 0 \quad \forall \zeta \neq 0, \zeta \in \mathcal{C}^\perp \tag{5}$$

for some vector subspace $\mathcal{C} \subset \mathbb{R}^q$, being $\mathcal{C}^\perp$ its orthogonal complement.

Proof is trivial, as symmetric matrices have an orthonormal basis of eigenvectors.

2.2 Takagi-Sugeno modelling

The well-known sector nonlinearity methodology rewrites the nonlinear expression on the right-hand side of (1) as an algebraically *equivalent* convex sum of linear models

$$\dot{x}(t) = \textstyle\sum_{i=1}^r h_i(x)\,(A_i x + B_i u)\,, \tag{6}$$

where the membership functions (MFs) h_i, grouped in a vector $h \in \mathbb{R}^r$, belong to the $r-1$-dimensional standard simplex

$$\Delta := \{h \in \mathbb{R}^r : \textstyle\sum_{i=1}^r h_i = 1, h_i \geq 0\ \forall i\},$$

provided the nonlinearities belong to a compact set of the state space Ω, including the origin.

Basically, sector nonlinearity methodology begins by taking the nonlinearities of (1), each of these, say $\rho_j(x)$, as in (Robles et al., 2015), is decomposed as follows:

$$\rho_j(x) = \frac{\rho_j(x)}{\eta_j(x)}\eta_j(x) := \tilde{\rho}_j(x)\eta_j(x), \quad j = 1, \ldots, s \tag{7}$$

where $\eta_j(x)$ is any *linear* function of the state such that $\eta_j(0) = 0$, thus enforcing that the limit of $\tilde{\rho}_j$ exists when $x \to 0$. The number of such nonlinearities has been denoted with s. Then, bounding $\rho_j(x)$ by two sectors

$$\left(\min_{x\in\Omega} \tilde{\rho}_j(x)\right)\eta_j(x) \leq \rho_j(x) \leq \left(\max_{x\in\Omega} \tilde{\rho}_j(x)\right)\eta_j(x) \tag{8}$$

means that the nonlinearity ρ_j can be expressed as an interpolation between the minimum and maximum value in the above expression, i.e., $\rho_j = w_j\overline{\tilde{\rho}_j} + (1-w_j)\underline{\tilde{\rho}_j}$, where:

$$\overline{\tilde{\rho}_j} = \max_{x\in\Omega} \tilde{\rho}_j(x), \quad \underline{\tilde{\rho}_j} = \min_{x\in\Omega} \tilde{\rho}_j(x). \tag{9}$$

Consider a vector $\rho(x) \in \mathbb{R}^s$ whose entries are the terms ρ_j defined above; then, every combination of maxima/minima of the s nonlinear terms defined as (8), when substituted in $f(x(t))$ and $g(x(t))$, produces A_i and B_i, respectively, $i \in \{1, 2, \ldots, r\}$, $r = 2^s$. Each of them corresponds to a MF h_i which is defined as the product of w_j and/or $1-w_j$, $j \in \{1, 2, \ldots, s\}$, according to the corresponding combination. For later developments, let us denote as $\mathcal{L}$ the ordered list of TS consequents $\mathcal{L} := \{L_1, L_2 \ldots, L_r\}$, where $L_i = \{A_i, B_i\}$.

Remark 1. Note that the freedom in choosing η_j for the nonlinearities in $f(x)$ is not present for those in $g(x)$. Indeed, considering nonlinearities of $g\,(x(t))\,u(t)$ in (1), when following the structure of (7), we notice that u can be considered already "extracted" as a right factor, taking the role of $u \equiv \eta_j$, so $\tilde{\rho}$ are also forcedly set as the nonlinear elements in $g\,(x(t))$. As there is no freedom in g, for simplicity, in the sequel this work will consider only systems in the form:

$$\dot{x} = f(x) + Bu \tag{10}$$

trying to produce a wise choice of the η_i in $f(x)$.