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Quadratic Design of Robust Controllers for Uncertain T-S Models with D-Stability Contraints

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Abstract: This paper deals with the robust $\mathcal{D}$-stabilization of uncertain Takagi-Sugeno (T-S) fuzzy systems. New Linear Matrix Linearity (LMI) conditions are proposed for the design of non Parallel-Distributed-Compensation (non-PDC) controllers with $\mathcal{D}$-stability constraints, i.e forcing the poles of each linear polytopes of the closed-loop T-S plant with model uncertainties to belong in a prescribed LMI region. The LMI conditions are obtained through the use of a quadratic Lyapunov function candidate and relaxed by the introduction of free weighting matrices. To illustrate the effectiveness of the proposed approach, the $\mathcal{D}$-stabilization of an academic example of a fourth-order uncertain T-S model is provided in simulation.

Keywords: Uncertain T-S Fuzzy models, $\mathcal{D}-$stability, Non-PDC controller design, Quadratic Lyapunov Functions, Linear Matrix Inequality (LMI), Robust control.

1. INTRODUCTION

A Takagi-Sugeno (T-S) model is constituted by a set of linear models blended together by convex nonlinear membership functions (Takagi and Sugeno, 1985). It may accurately represent a nonlinear system in a compact set of its state space by using, for instance, sector nonlinearity transformations (Tanaka and Wang, 2001).

The stability and stabilization issues of T-S models are often investigated using the direct Lyapunov method, which aims to obtain Linear Matrix Inequality (LMI) conditions (Wang et al., 1996; Tanaka et al., 2003; Boyd et al., 1994). Such LMI conditions have been firstly obtained through common quadratic Lyapunov functions (QLF) involving a Parallel Distributed Compensation (PDC) control scheme (Tanaka and Wang, 2001; Wang et al., 1996). However, they require to find common decision variables, solution of a set of LMI, leading to conservatism (Sala, 2009). In order to reduce the conservatism, piecewise, switched, polynomial or non-quadratic Lyapunov functions have been proposed (Johansson et al., 1999; Tanaka et al., 2003; Guerra and Vermeiren, 2004; Ohtake et al., 2006; Jabri et al., 2012; Guelton et al., 2013). Nevertheless, some drawbacks occur. For instance, when using non-quadratic fuzzy Lyapunov functions in the continuous time case (Tanaka et al., 2003), the appearance of time-derivatives in the stability conditions makes harder the obtention of LMIs or their practical application (Guerra et al., 2012; Duong et al., 2013). To circumvent this problem, a mix between basic quadratic approaches and non-quadratic ones has been proposed, first by employing descriptor technics (Guerra et al., 2007; Seddiki et al., 2010; Bouarar et al., 2013), then by applying the Finsler's lemma (Jaadari et al., 2012). It consists on an extended quadratic approach where free fuzzy matrices are introduced as slack decision variables.

The improvement of the transient response of complex nonlinear systems represented by T-S models is the main objective of this paper. Nevertheless, an exact model may be difficult to obtain in practice since, during the modeling phase, uncertain parameters appear naturally (parametric uncertainty, imprecision of sensor,...). Generally, there are two approaches to deal with uncertainty. The first one is to consider the parameters as realizations of random processes and the second one takes into account the bound intervals characterizing each uncertainty (Zhou and Khargonekar, 1988). Then, to improve the closed-loop performances of the T-S plant, $\mathcal{D}$-stability concept will be considered (Chilali et al., 1999). This consists in imposing new constraints on the Lyapunov function in order to force the poles of the closed-loop linear polytopes to belong in a prescribed LMI region (Bachelier, 1998; Peaucelle et al., 2000). Following our previous works dealing with fuzzy controller design under $\mathcal{D}$-stability constraints (Cherifi et al., 2015a,b), the goal of this paper is to propose new relaxed LMI conditions for robust controller design, which ensure both the uncertain closed-loop stability and some prescribed performances.

This paper is organized by the following guideline. After presenting some useful preliminaries, one derives standard robust quadratic conditions for uncertain T-S closed-loop system including $\mathcal{D}$-stability constraint. Then relaxed quadratic robust $\mathcal{D}$-stability conditions are proposed for the design of non-PDC controllers. Finally, the effectiveness of the proposed results is illustrated through a numerical example.

2. PRELIMINARIES

Consider the following uncertain T-S model given by (Tanaka and Wang, 2001):

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$$\dot{x}(t) = \sum_{i=1}^{r} h_i(\xi(t))(\tilde{A}_i(t)x(t) + \tilde{B}_i(t)u(t)) \tag{1}$$

where r is the number of vertices, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $\xi(t) \in \mathbb{R}^p$ is the vector of premises, $h_i(\xi(t)) \in [0,1]$ are convex membership functions satisfying $\sum_{i=1}^{r} h_i(\xi(t)) = 1$, and:

$$\tilde{A}_i(t) = A_i + \Delta A_i(t) \in \mathbb{R}^{n\times n} \tag{2}$$

and

$$\tilde{B}_i(t) = B_i + \Delta B_i(t) \in \mathbb{R}^{n\times m} \tag{3}$$

where A_i and B_i are real constant matrices defining the i^{th} nominal vertex, $\Delta A_i(t)$ and $\Delta B_i(t)$ are Lesbesgue measurable bounded uncertainties, which can be rewritten as follows (Zhou and Khargonekar, 1988):

$$\Delta A_i(t) = E_{ai}\delta_a(t)L_{ai} \tag{4}$$

and

$$\Delta B_i(t) = E_{bi}\delta_b(t)L_{bi} \tag{5}$$

where E_{ai}, E_{bi}, L_{ai}, L_{bi} are real constant matrices with appropriate dimension and $\delta_a(t)$ and $\delta_b(t)$ are uncertain vectors which verify the bounded conditions:

$$\delta_a^T(t)\delta_a(t) \leqslant I \tag{6}$$

and

$$\delta_b^T(t)\delta_b(t) \leqslant I \tag{7}$$

In order to provide relaxed controller design, let us consider the non-PDC control law given by (Jaadari et al., 2012):

$$u(t) = \sum_{i=1}^{r} h_i(\xi(t))F_i \left(\sum_{j=1}^{r} h_j(\xi(t))H_j\right)^{-1} x(t) \tag{8}$$

where $F_i \in \mathbb{R}^{m\times n}$ and $H_j \in \mathbb{R}^{n\times n}$ are constant gain matrices to be synthesized.

Notations: In the sequel, the time t will be omitted in mathematical expressions when there is no ambiguity. An asterisk $(*)$ denotes a transpose quantity in a matrix and, for any real square matrices R, $\mathcal{H}(R) = R + R^T$. Consider a set of real matrices M_i and N_{ij}, for all $(i,j) \in \{1,...,r\}^2$, one denotes $M_h = \sum_{i=1}^{r} h_i(\xi)M_i$, $N_{hh} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\xi)h_j(\xi)N_{ij}$ and $H_h^{-1} = \left(\sum_{j=1}^{r} h_j(\xi(t))H_j\right)^{-1}$.

From (1) and (8), the closed-loop dynamics may be expressed as:

$$\dot{x}(t) = (\underbrace{G_{hh} + \Delta G_{hh}}_{\tilde{G}_{hh}})x(t) \tag{9}$$

where $G_{hh} = A_h + B_hF_hH_h^{-1}$ and $\Delta G_{hh} = \Delta A_h(t) + \Delta B_h(t)F_hH_h^{-1}$.

The aim of this work is to propose new LMI based conditions allowing to design a robust non-PDC control law (8) such that the closed-loop dynamics (9) is $\mathcal{D}$−stable whatever $\delta_a(t)$ and $\delta_b(t)$ are with respect to (6) and (7). This will be achieved thanks to the following lemmas and definitions.

Lemma 1. (Xie and De Souza, 1992): Let X and Y be real matrices of appropriate dimensions. For any real positive definite matrix $T = T^T > 0$, the following inequality hold:

$$\mathcal{H}(X^TY) \leq X^TTX + Y^TT^{-1}Y \tag{10}$$

Lemma 2. Schur complement (Boyd et al., 1994): Let M, R and S be matrices of appropriate dimensions and S invertible. The following statements are equivalent:

$$M - R^TSR < 0, \text{ with } S < 0 \tag{11}$$

$$\begin{bmatrix} M & R^T \\ R & S^{-1} \end{bmatrix} < 0 \tag{12}$$

Lemma 3. (Tuan et al., 2001): Let Γ_{ij}, $(i,j) \in \{1,...,r\}^2$, be matrices of appropriate dimensions. $\Gamma_{hh} < 0$ is satisfied if both the following conditions hold:

$$\Gamma_{ii} < 0, \forall i \in \{1,...,r\} \tag{13}$$

$$\frac{2}{r-1}\Gamma_{ii} + \Gamma_{ij} + \Gamma_{ji} < 0, \forall (i,j) \in \{1,...,r\}^2 / i \neq j \tag{14}$$

Note that several relaxation schemes have been proposed in the context of T-S model-based design (Sala, 2009). However, lemma 3 will be considered in the sequel since it is reputed as a good compromise between complexity and conservatism improvement.

The following useful definitions are provided to deal with the $\mathcal{D}$-stability concepts (Chilali et al., 1999).

Definition 1. (Chilali et al., 1999). A subset $\mathcal{D}$ of the complex plane is called an LMI region if it is defined by the matrices $L = L^T \in \mathbb{R}^{d\times d}$ and $M \in \mathbb{R}^{d\times d}$ such that:

$$\mathcal{D} = \{\lambda \in \mathbb{C} : L + \lambda M + \lambda M^T < 0\} \tag{15}$$

where d is called the order of the LMI region.

In the sequel, we will consider the LMI region depicted in figure 1 and defined by:

(1) the left half plan defined by $\mathcal{R}e(\lambda) < \beta$,
(2) a conic sector defined by its apex at $(\gamma, 0)$ and an inner angle $\pi/2 - \theta$,
(3) a circle centered at $(q, 0)$ with a radius s.

With regards to definition 1, this leads to the following matrices (Chilali et al., 1999):

$$L = \begin{bmatrix} -2\beta & 0 & 0 & 0 & 0 \\ 0 & -2\gamma\cos\theta & 0 & 0 & 0 \\ 0 & 0 & -2\gamma\cos\theta & 0 & 0 \\ 0 & 0 & 0 & -s & -q \\ 0 & 0 & 0 & -q & -s \end{bmatrix} \tag{16}$$

and

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 & 0 \\ 0 & -\sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{17}$$

For more details and examples illustrating how the matrices L and M are set for different LMI regions, the reader may consult (Chilali et al., 1999; Bachelier, 1998).

The following definition expresses the basic $\mathcal{D}$-stability conditions for nonlinear models with regards to the Lyapunov theory.

Definition 2. (Bai et al., 2015). Given an LMI region defined by (15), a nonlinear system $\dot{x} = f(x)x$ is said to