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Nonfragile H_∞ Filtering for Discrete-Time Nonlinear Interconnected Systems

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Abstract: This paper is concerned with the non-fragile H_∞ filtering problem for a class of discrete-time nonlinear interconnected systems with Takagi-Sugeno (T-S) fuzzy model. Attention is focus on the design of a fuzzy filter such that the filtering error system preserves a prescribed H_∞ performance, and the filter to be designed is assumed to have gain variations. Sufficient conditions are obtained to ensure that the filtering error system is asymptotically stable with a prescribed H_∞ performance level based on the robust control approach. A simulation example is given to show the efficiency of the proposed methods.

Keywords: Takagi-Sugeno (T-S) fuzzy systems, H_∞ filtering, linear matrix inequalities (LMIs), non-fragile, nonlinear interconnected system.

1. INTRODUCTION

With further research, it's known that the mathematical models of present-day physical, engineering, and societal systems are frequently of high dimension or possess interacting dynamic phenomena. The methodologies of large-scale interconnected systems provide techniques through the manipulation of a system structure given a good description of this problem. There were considerable interests in the research of large-scale interconnected systems in the past years (Siuak (1978)). Many methods have been developed to investigate the stability analysis and control design of large-scale interconnected systems (Ioannou et al. (1986)).

On the other hand, there has been rapidly growing interest in filtering of nonlinear systems using the Takagi-Sugeno (T-S) fuzzy model (Takagi and Sugeno (1985)) In the past several decades. And much effort has been devoted to the H_∞ filtering for T-S fuzzy systems. The advantage of using H_∞ filtering in comparison with the traditional Kalman filtering is that no statistical assumptions on the exogenous signals are needed. Recently, the stability analysis and stabilization of fuzzy large-scale interconnected systems were discussed in (Wang and Lin (2005)). H_∞ filtering of fuzzy large-scale interconnected systems was discussed in (Zhang et al. (2010)). Chang and Yang (Zhang et al. (2010)) were concerned with the problem of fuzzy non-fragile H_∞ filtering, in which the fuzzy filter to be designed is assumed to have gain variations. Because A filter inevitably exist the uncertain factors in the implementation, such as the ageing of the internal components, external temperature fluctuations. Unavoidably the filter coefficient will change. Despite the fact that there have been fruitful results on H_∞ filtering for T-S fuzzy systems, these results do not address the problem of designing nonfragile filters for fuzzy large-scale interconnected systems.

In this paper, we study the problem of non-fragile H_∞ filtering for discrete-time nonlinear interconnected systems based on the T-S fuzzy model. The focus is on designing a fuzzy filter with the gain variations such that the filtering error system guarantees a prescribed H_∞ performance level. A descriptor representation approach is developed to eliminate the coupling between the system matrices and the designed filter matrices. It shows that the solution of the design problem can be obtained by solving a set of LMIs. Finally, a simulation example is provided to illustrate the feasibility of the proposed design methods.

2. PRELIMINARIES AND PROBLEM FORMULATION

Suppose there is an interconnected system which composed of J subsystem S_i, $i = 1, 2, \dots, J$. Then the T-S fuzzy model which represent the subsystem S_i by fuzzy IF-THEN rules is shown as below:

Rule j: IF $\theta_{i1}(t)$ is F_{i1j} and ... and θ_{ig} is F_{igj} THEN

$$S_i : \begin{cases} x_i(k+1) = A_{ij}x_i(k) + B_{ij}w_i(k) \\ \qquad + \sum\limits_{m=1, m\neq i}^{J} C_{im}x_m(k) \\ y_i(k) = D_{ij}x_i(k) + E_{ij}v_i(k) \\ z_i(k) = H_{ij}x_i(k) \\ x_i(k) = \phi_i(k), k = 0, -1, ..., -k \end{cases} \tag{1}$$

where $j \in L_i = \{1, 2, \dots, r_i\}$ denotes the jth fuzzy inference rule; r_i is the number of inference rules; $\theta_i(k) = (\theta_{i1}(k), \theta_{i2}(k), \dots, \theta_{ig}(k)) \in R^{n_{ig}}$ are some measurable premise variables; and $F_{ipj} (p = 1, 2, \dots, g)$ are fuzzy sets;

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$x_i(k) \in R^{n_{ix}}$ is the state vector; $y_i(k) \in R^{n_{iy}}$ is the output vector; $z_i(k) \in R^{n_{iz}}$ is the signal vector to be estimated; $w_i(k) \in R^{n_{iw}}$ is the disturbance input vector that is assumed to belong to $l_2[0,\infty)$; $(A_{ij}, B_{ij}, D_{ij}, E_{ij}, H_{ij})$ represent the jth model of the ith fuzzy subsystem; C_{im} denotes the interconnected between the ith and mth subsystem, and $C_{im}=0$ for $i=m$; $\phi_i(t)$ the given initial condition sequence. with

$$\mu_{ij}(\theta_i(t)) = \prod_{p=1}^{g} F_{ipj}(\theta_{ip}(k))$$

$$h_{ij}(\theta_i(k)) = \frac{\mu_{ij}(\theta_i(k))}{\sum\limits_{j=1}^{r_i} \mu_{ij}(\theta_i(k))}$$

where $F_{ipj}(\theta_{ip}(k))$ is the grade of membership of $\theta_{ip}(k)$ in F_{ipj}. It is assumed that $\mu_{ij}(\theta_i(k)) \geq 0$, and we know $h_{ij}(\theta_i(k)) \geq 0$, $\sum\limits_{j=1}^{r_i} \mu_{ij}(\theta_i(k)) = 1$ for all t, where $j = 1,2,\ldots,r_i$, $i=1,2,\ldots,J$.

In this paper,we consider the following non-fragile filter for the fuzzy subsystem S_i:

Filter rule j: IF $\theta_{i1}(t)$ is F_{i1j} and ... and θ_{ig} is F_{igj} THEN

$$S_{fi}: \begin{cases} x_{fi}(k+1) = (A_{fij} + \Delta A_{fij}) x_{fi}(k) + (B_{fij} \\ \qquad + \Delta B_{fij}) y_i(k) \\ z_{fi}(k) = (C_{fij} + \Delta C_{fij}) x_{fi}(k) \\ x_{fi}(k) = \phi_{fi}(k), k = 0, -1, \ldots, -k \end{cases} \quad (2)$$

where $x_{fi}(k)$ is the state variable of the filter, $z_{fi}(k)$ is an estimation of $z_i(k)$ and A_{fij}, B_{fij}, C_{fij} are filter parameter matrices to be determined. ΔA_{fij}, ΔB_{fij} and ΔC_{fij} represent the following interval type of gain varitions:

$$\begin{cases} \Delta A_{fij} = M_{1ij} \Delta_{1ij}(k) N_{1ij} \\ \Delta B_{fij} = M_{2ij} \Delta_{2ij}(k) N_{2ij} \\ \Delta C_{fij} = M_{3ij} \Delta_{3ij}(k) N_{3ij} \end{cases} \quad (3)$$

In the above perturbations, matrices M_{1ij}, M_{2ij}, M_{3ij}, N_{1ij}, N_{2ij}, N_{3ij} are known with appropriate dimension. $\Delta_{1ij}(k)$, $\Delta_{1ij}(k)$, and $\Delta_{1ij}(k)$ are unknown continuous functions and satisfying

$$\begin{array}{l} \Delta_{1ij}^T(k) \Delta_{1ij}(k) \leq I \\ \Delta_{2ij}^T(k) \Delta_{2ij}(k) \leq I \\ \Delta_{3ij}^T(k) \Delta_{3ij}(k) \leq I. \end{array} \quad (4)$$

Then, we can abtain the filtering-error subsystem as belows:

$$\Omega_i: \begin{cases} \tilde{x}_i(k+1) = \bar{A}_i(h) \tilde{x}_i(k) + \bar{B}_i(h) w_i(k) \\ \qquad + \sum\limits_{m=1, m\neq i}^{J} \bar{C}_{im} \tilde{x}_m \\ \tilde{z}_i(k) = \bar{H}_i(h) \tilde{x}_i(k). \end{cases} \quad (5)$$

By defining a new state vector $\tilde{x}_i(k) = \left[x_i^T(k), x_{fi}^T(k)\right]^T$, and the estimated error vector $\tilde{z}_i(k) = z_i(k) - z_{fi}(k)$. The uncertainties in the filter gain are of the form

$$\begin{array}{l} \bar{A}_i(h) = \tilde{A}_i(h) + \Delta\tilde{A}_i(h) = \tilde{A}_i(h) + \widetilde{M}_{1i} \widetilde{\Delta}_1(k) \widetilde{N}_{1i} \\ \bar{B}_i(h) = \tilde{B}_i(h) + \Delta\tilde{B}_i(h) = \tilde{B}_i(h) + \widetilde{M}_{2i} \widetilde{\Delta}_2(k) \widetilde{N}_{2i} \\ \bar{H}_i(h) = \tilde{H}_i(h) + \Delta\tilde{H}_i(h) = \tilde{H}_i(h) + \widetilde{M}_{3i} \widetilde{\Delta}_3(k) \widetilde{N}_{3i} \end{array} \quad (6)$$

$$\begin{array}{l}
\tilde{A}_i(h) = \begin{bmatrix} A_i(h) & 0 \\ B_{fi}(h) D_i(h) & A_{fi}(h) \end{bmatrix} = \\
\sum\limits_{j=1}^{r_i} \sum\limits_{k=1}^{r_i} h_{ij}(\theta_i(k)) h_{ik}(\theta_i(k)) \begin{bmatrix} A_{ij} & 0 \\ B_{fik} D_{ij} & A_{fik} \end{bmatrix} \\
\tilde{B}_i(h) = \begin{bmatrix} B_i(h) & 0 \\ 0 & B_{fi}(h) E_i(h) \end{bmatrix} = \\
\sum\limits_{j=1}^{r_i} \sum\limits_{k=1}^{r_i} h_{ij}(\theta_i(k)) h_{ik}(\theta_i(k)) \begin{bmatrix} B_{ij} & 0 \\ 0 & B_{fik} E_{ij} \end{bmatrix} \\
\tilde{H}_i(h) = [\, H_i(h) \ \ -C_{fi}(h) \,] = \\
\sum\limits_{j=1}^{r_i} \sum\limits_{k=1}^{r_i} h_{ij}(\theta_i(k)) h_{ik}(\theta_i(k)) [\, H_{ij} \ \ -C_{fik} \,] \\
\bar{C}_{im} = \begin{bmatrix} C_{im} & 0 \\ 0 & 0 \end{bmatrix} \\
\widetilde{M}_{1i} = \begin{bmatrix} 0 & 0 \\ M_{2i} & M_{1i} \end{bmatrix}, \widetilde{M}_{2i} = \begin{bmatrix} 0 \\ M_{2i} \end{bmatrix} \\
\widetilde{M}_{3i} = [\, 0 \ \ M_{3i} \,] \\
\widetilde{N}_{1i} = \begin{bmatrix} N_{2i} D_i(h) & 0 \\ 0 & N_{1i} \end{bmatrix}, \widetilde{N}_{2i} = \begin{bmatrix} 0 \\ N_{2i} E_i(h) \end{bmatrix} \\
\widetilde{N}_{3i} = [\, 0 \ \ -N_{3i} \,] \\
\widetilde{\Delta}_{1i} = \begin{bmatrix} \Delta_{2i} & 0 \\ 0 & \Delta_{1i} \end{bmatrix}, \widetilde{\Delta}_{2i} = [\, 0 \ \ \Delta_{2i} \,] \\
\widetilde{\Delta}_{3i} = \begin{bmatrix} 0 \\ \Delta_{3i} \end{bmatrix}.
\end{array} \quad (7)$$

Let $\tilde{z}(k) = \left[\tilde{z}_1^T(k), \tilde{z}_2^T(k), \ldots, \tilde{z}_J^T(k)\right]^T$. The non-fragile H_∞ filtering problem of this system to be addressed in this paper can be formulated as follows.

Given a fuzzy system (1) and a prescribed level of noise attenuation $\gamma > 0$, find a filter in the form of (2) such that the filtering-error system composed of J filtering-error subsystems as (5) is asymptotically stable, and the following overall H_∞ performance is satisfied

$$\|\tilde{z}(k)\|_2 < \gamma \|\tilde{w}(k)\|_2 \quad (8)$$

for all nonzero $w(k) \in l_2[0, \infty)$.

The following lemma is needed in the latter development of the main results.

lemma 1 (Fu and Xie (2005)). For given appropriate matrices Ξ_1, Ξ_2, Ξ_3 with $\Xi_1 = \Xi_1^T$,

$$\Xi_1 + \Xi_3 \widehat{\Delta}(k) \Xi_2 + \Xi_2^T \widehat{\Delta}^T(k) \Xi_3^T < 0 \quad (9)$$

holds for all $\widehat{\Delta}(k)$ satisfying $\widehat{\Delta}^T(k) \widehat{\Delta}(k) \leq I$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$\Xi_1 + \varepsilon \Xi_3 \Xi_3^T + \varepsilon^{-1} \Xi_2^T \Xi_2 < 0. \quad (10)$$

lemma 2. Let $\Xi_1 = \Xi_1^T$, Ξ_{2ij}, Ξ_{3ij} for i, $j$and $\Delta(k)$ be real matrices with appropriate dimension and $\Delta^T(k)\Delta(k) \leq I$. Then, for any scalar $\varepsilon > 0$

$$\begin{array}{l}
\Xi_1 + \sum\limits_{i=1}^{r} \sum\limits_{j=1}^{r} h_i(\xi(k)) h_j(\xi(k)) (\Xi_{2ij} \Delta(k) \Xi_{3ij} \\
\quad + \Xi_{2ij}^T \Delta(k) \Xi_{3ij}^T) \leq \Xi_1 \\
+ \sum\limits_{i=1}^{r} \sum\limits_{j=1}^{r} h_i(\xi(k)) h_j(\xi(k)) \left(\frac{1}{\varepsilon_{ij}} \Xi_{2ij} \Xi_{2ij}^T \right. \\
\quad \left. + \varepsilon_{ij} \Xi_{3ij}^T \Xi_{3ij} \right).
\end{array} \quad (11)$$