Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 49-5 (2016) 031–036

A comparison of fuzzy identification methods on benchmark datasets

Darko Aleksovski * Dejan Dovžan ** Sašo Džeroski * Juš Kocijan ***

* *Department of Knowledge Technologies, Jožef Stefan Institute, Jamova cesta 39, Ljubljana, Slovenia*
** *Faculty of Electrical Engineering, University of Ljubljana, Tržaška cesta 25, Ljubljana, Slovenia*
*** *Department of Systems and Control, Jožef Stefan Institute, Jamova cesta 39, Ljubljana, Slovenia, and University of Nova Gorica, Vipavska 13, Nova Gorica, Slovenia (e-mail: jus.kocijan@ijs.si)*

Abstract: In this paper, we address the task of discrete-time modeling of nonlinear dynamic systems. We use Takagi-Sugeno fuzzy models built by LOLIMOT and SUHICLUST, as well as ensembles of LOLIMOT fuzzy models to accurately model nonlinear dynamic systems from input-output data. We evaluate these approaches on benchmark datasets for three laboratory processes. The measured data for the case studies are publicly available and are used for development, testing and benchmarking of system identification algorithms for nonlinear dynamic systems. Our experimental results show that SUHICLUST produces smaller models than LOLIMOT for two of the three datasets. In terms of error, ensembles of LOLIMOT models improve the predictive performance over that of a single LOLIMOT or SUHICLUST model.

Keywords: Fuzzy model identification, tree partitioning, fuzzy clustering, ensembles

1. INTRODUCTION

This paper considers the task of discrete-time modeling of nonlinear dynamic systems from input-output data. Using the *external dynamics approach* (Nelles, 2001), the task of empirical modeling of a dynamic system can be formulated as a regression problem of finding a difference equation that fits an observed behavior of the system.

One possible approach to the regression (i.e., nonlinear function approximation) problem is the *multimodel approach* (Murray-Smith and Johansen, 1997). Its idea is to combine several simple submodels and use them to describe the global behavior of the dynamic system. The operating region of the dynamic system is split into several subregions (partitions) and a simple local model is built for each subregion. The local models are combined by smooth interpolation into a complete global model.

Two fuzzy identification methods are considered, which are based on the multimodel approach. These methods determine the subregions by utilizing an automatic procedure which uses input-output data. In the literature, the problem of automatic defining of operating regions is solved using grid-based (Jang et al., 1997), fuzzy clustering (Jang et al., 1997), or tree partitioning methods (Nelles, 2001; Jang, 1994). The fuzzy methods evaluated here follow the last two principles.

Also, a local model is built for each identified region. Common approaches to parameter estimation in the local models are local or global optimization, frequently used with the least-squares cost function (Johansen and Babuška, 2003). The first estimates the parameters in each local model separately, while the second estimates the parameters in all local models jointly. The local optimization, which is faster and can handle noisy data, is employed in the fuzzy methods considered here.

This paper compares neuro-fuzzy identification methods, based on, or derived from the Lolimot method. In particular, it is concerned with building Takagi-Sugeno (TS) models (Takagi and Sugeno, 1985), by using LOLIMOT (Nelles, 2001) and SUHICLUST (Hartmann et al., 2013). Additionally, it considers the more accurate ensembles of fuzzy models (Aleksovski et al., 2015). The ensembles are built by using the bagging procedure and use LOLIMOT fuzzy models as base models. However, this paper does not include a comparison to adaptive or evolving methods because they are not of the same sort. Adaptive and evolving methods change their parameters with time and in the case of evolving methods also structure, while the models evaluated here are invariant to changes.

The LOLIMOT method (Nelles, 2001) uses an incremental construction scheme, and builds TS fuzzy models. It uses a tree partitioning method for defining the operating regions, i.e. for model structure identification. For the local models it uses local least squares estimation. The method is fast and noise-tolerant, due to the use of local estimation. The tree partitioning allows it to handle large dimensional problems efficiently: partitions have different sizes and are located in regions where the data allow for a finer partitioning. However, the partitioning is axis-

2405-8963 © 2016, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2016.07.085

parallel: the partitions are hyperrectangles whose sides are parallel with the axes of the input space.

The SUHICLUST method was introduced by Hartmann et al. (2013). It unifies the strengths of supervised, incremental tree construction scheme of LOLIMOT with the advantages of product space clustering. By merging these two concepts, a fuzzy identification algorithm is obtained that, in contrast to LOLIMOT, enables axes-oblique partitioning and has highly flexible validity functions. According to Hartmann et al. (2011), the method produces fuzzy models with the same accuracy as LOLIMOT, but with fewer local models. The reproducibility of results is the same as with LOLIMOT and therefore better than with product space clustering. In Teslič et al. (2011) the SUHICLUST method was successfully used to model the drug absorption spectra process. Recently, an approach for the design of experiments based on the SUHICLUST method was proposed by Škrjanc (2015).

Ensembles (Krogh and Vedelsby, 1995; Dietterich, 2002), or committees of predictors, work by creating several *base models*. Each base model is an imperfect predictive model capturing a potentially different aspect of the system being modeled. Combining the imperfect predictions obtained from each base model should improve the predictive power over a single model and thus increase the accuraccy.

Ensembles based on the bootstrap resampling principle (Breiman, 1996) identify each base model from a modified version of the training data. First, several bootstrap samples are created from the training data, after which a model is built for each sample. The same principle is used by Aleksovski et al. (2015) for building ensembles of LOLIMOT models both for single-output and multi-output nonlinear dynamical systems.

The remainder of the paper is organized as follows. The next section describes the methodology: the use of LOLIMOT to build (single) fuzzy models and ensembles, and the use of SUHICLUST. Section 3 presents the experimental setup and describes the case studies, while Section 4 presents the results of the experimental evaluation. Finally, Section 5 concludes and outlines some directions for further work.

2. METHODOLOGY

This section introduces the fuzzy methods LOLIMOT and SUHICLUST, and describes the procedure for building ensembles of fuzzy models by using LOLIMOT. The fuzzy models evaluated in this paper have the Takagi- Sugeno (TS) form:

$$\hat{\boldsymbol{y}}(\boldsymbol{x}) = \sum_{j=1}^{m} \Phi_j(\boldsymbol{x}) f_{LMj}(\boldsymbol{x}). \quad (1)$$

Similarities between LOLIMOT and SUHICLUST. The methods evaluated in this paper use two different (automatic) procedures for defining operating regions (partitions), based on identification data. LOLIMOT uses tree partitioning, while SUHICLUST uses fuzzy clustering. The tree partitioning used in LOLIMOT is a computationally efficient approach which can handle large dimensional problems: it is able to create partitions of different sizes, which are smaller in those parts of the input space where finer partitioning is needed. Simpler grid partitioning methods suffer from the curse of dimensionality, as they create a complete grid over the input space. The second method evaluated here, SUHICLUST, defines the operating regions using a combination of fuzzy clustering, with the Gustafson-Kessel algorithm, and supervised learning (Hartmann et al., 2011).

2.1 LOLIMOT

The local linear model trees (LOLIMOT) method (Nelles, 2001) operates iteratively, using a tree partitioning procedure. In each iteration, it selects the worst performing partition, and splits it further. Local models are estimated at the end of each iteration for the new partitions.

In particular, each partition obtains a multi-dimensional Gaussian fuzzy membership function $\Phi_j(\boldsymbol{x})$, which is used to estimate a local affine model f_{LM_j}. Finally, the resulting model is a TS model where the output is calculated as:

$$\hat{y}(\boldsymbol{x}) = \sum_{j=1}^{m} (b_{j,0} + b_{j,1}x_1 + b_{j,2}x_2 + ... + b_{j,p}x_p)\Phi_j(\boldsymbol{x}), \quad (2)$$

and m is the number of local models.

Selection heuristic. In each iteration, LOLIMOT considers only one partition for further expansion. It selects the worst performing one, using the criterion of largest sum of squared errors. As LOLIMOT was designed for dynamic system identification, it is able to use simulation to evaluate the intermediate model in each step. It is performed using all available training data, and no averaging is used on the individual squared errors. [1]

After determining the worst performing partition, LOLIMOT considers splitting the partition in half, in every possible dimension. Each of these alternatives is evaluated by using a heuristic greedy evaluation function: (a) the partition boundaries are adjusted, and a fuzzy model with one more LM is created, (b) two new local models are estimated, and (c) simulation is used to evaluate the fit od the complete model to the training data.

Estimation of local model parameters. In each iteration of the method, the parameters of the newly added local models are estimated. The estimation begins by calculating the fuzzy membership function values. As a next step, these values are used in the weighted least square regression performed to obtain the parameters of the local models.

The membership functions determine the (fuzzy) membership of each data point to each of the partitions and the corresponding local models. LOLIMOT uses the multi-dimensional Gaussian membership function, whose center $\mathbf{c}$ is determined by the center of the partition, and standard deviation vector $\boldsymbol{\sigma}$ is calculated as 1/3 of the size of the partition (Nelles, 2001). Thus, the membership of a data point $\mathbf{x}$ to the j-th partition is calculated as

$$\mu_j(\mathbf{x}) = exp(-\frac{1}{2}\sum_{i=1}^{n}(\frac{x_i - c_i}{\sigma_i})^2). \quad (3)$$

[1] It is interesting to note that such a selection heuristic produces more partitions in the regions which contain more training data.