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Model predictive control of partially fading memory systems with binary inputs

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ABSTRACT

This paper presents a heuristic algorithm to implement a model predictive controller for systems with binary inputs in which the effect of the control signal on the response partially vanishes before reaching steady state, for example systems that exhibit both fast and slow stable dynamics. The proposed algorithm is based on an iterative procedure that constructs a reduced set of suboptimal solutions. The size of this set can be set accordingly to the computing capabilities and the sample time. The iterative procedure rejects possible solutions profiting from the partial fading memory property of the system and an approximation of the optimal cost-to-go function. The properties of the proposed controller are illustrated with a simulated example of a photobioreactor.

1. Introduction

Model predictive control (MPC) [7] is one of the most successful forms of feedback control due to its ability to control almost any kind of process while considering operating constraints explicitly. The theoretical bases have been well established for both linear and nonlinear systems [17] and, in many cases, a real time implementation is not a problem. These are some of the reasons that explain why MPC is more spread in the industry than any other form of modern control. There are, however, cases in which the real time application can be difficult due to the computational burden associated with the computation of the control law, specially when the model is based on a mix of integer and real variables [8,3]. A particular case of this scenario will be considered in this paper.

The control action considered in most MPC strategies can take any value from a prescribed range. However, there are systems in which the control action is just a binary value. This is the case when an on–off actuator is used. Such actuators appear in the form of on–off valves, pulse width modulation switches, power electronic devices or thrusters in spacecrafts, to name a few examples. Having the control action restricted to a set of discrete values makes the optimization problem associated to MPC much harder to solve. Instead of a relatively benign quadratic or linear programming problem, a much more complex mixed integer programming problem must be solved in real time each sampling interval. Thus, efficient optimization algorithms are very important in these cases.

In order to solve a binary optimization problem it is necessary in general, to compute all the possible combinations of the control actions along the prediction horizon and select the one with the lowest cost. As the number of combinations grows exponentially with the length of the prediction horizon, this can only be done if the prediction horizon is small as in Sprock and Hsu [21], Ghanes et al. [12], where MPC of switched power electronics is considered. When the prediction horizon is large, the number of candidate solutions must be reduced by some strategy [22].

In Causa et al. [9] two different approaches based on branch and bound techniques and genetic algorithms were applied to the control of a batch reactor with on–off valves. Genetic algorithms have also been used by Schmitz et al. [20] for the control of the aeration in a waste water treatment plant with the goal of lowering the operating costs. Pawlowski et al. [16] and Berenguel et al. [4] used branch and bound algorithms together with an event driven sampling mechanism to apply an MPC strategy to the control of pH in photobioreactors. Attitude control of a spacecraft using on–off thrusters with linear parameter varying models and branch and bound techniques has been considered in Asakawa and Kida [2]. The solution of the MIQP problem using methods not based on branch and bound has also been considered in the context of opti-

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mal control problems of systems with discrete inputs. Sager et al. [19] used, in the optimal control of a subway train with discrete gears, a convexification on the control inputs and a relaxation of the MIQP problem in a proposed strategy based on the direct multiple shooting method [5]. Finally in some rare cases, the optimal control policy can be identified a priori and online optimization can be avoided [18].

This paper presents a heuristic algorithm to obtain a suboptimal solution of the MPC optimization problem for processes with binary inputs in which a part of the dynamics is not influenced by past values of the control signal beyond a certain time instant. We denote this class of systems as partial fading memory systems (PFMS). The algorithm constructs iteratively a series of sets of candidate solutions that are likely to contain the optimal solution, in a similar way to the L-Band algorithm [1,11], profiting from the partial fading memory property of the system and an approximation of the optimal cost-to-go function. The size of this set provides a trade-off between optimality and computational burden. These properties are illustrated by means of a simulated example.

The paper is organized as follows: the problem formulation is presented in Section 2. The main contributions of the paper are presented in Section 3. An example of application is shown is Section 4. The paper ends with the conclusions in Section 5.

2. Problem formulation

The system considered throughout the paper will be represented by a discrete-time linear model:

$$x(t+1) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ is the state of the process, $u(t) \in \{0, 1\}$ is the binary control input, t is discrete valued, $A \in \mathbb{R}^{n_x \times n_x}$ is the state transition matrix and $B \in \mathbb{R}^{n_x}$ is the input matrix.

Definition 1 *(Partially fading memory system).* A system is said to be a partially fading memory system if for any given state sequences $x_a(t), x_b(t)$ with different initial states but driven by the same input sequence, the following holds:

$$\|x_a(t) - x_b(t)\| \le \|C_s(x_a(t) - x_b(t))\| + \sigma_f^t \|C_f(x_a(0) - x_b(0))\|, \ \forall t \ge L$$

where $L \in \mathbb{N}$ is the fading time of the fast dynamics, $\sigma_f \in [0, \ 1)$ is the fading parameter such that $\sigma_f^L \ll 1$ and C_s and C_f are projection matrices that satisfy $\|x_a(t) - x_b(t)\| \le \| C_s(x_a(t) - x_b(t)) \| + \| C_f(x_a(t) - x_b(t)) \| \quad \forall t$, where $\|\cdot\|$ is a given vector norm.

This definition implies that part of the state $x(t)$ for time instant t is only barely influenced by past values of $u(t)$ beyond $t - L$ (i.e, $u(t - L), u(t - L - 1), \ldots$). This can be seen as the fast dynamics part of the state, denoted as $z^f(t)$ so that

$$z^f(t) = C_f x(t).$$

On the other hand, the remaining part of the state is influenced by past values of $u(t)$ beyond $u(t - L)$. This part of the state vector, the slow dynamics of system (1), will be denoted as $z^s(t)$, so that

$$z^s(t) = C_s x(t).$$

Systems that combine stable and integrating dynamics and systems with both fast and slow stable dynamics are examples of PFMS.

Assumption 1. System (1) is a partially fading memory system (PFMS).

Assuming that (1) is a partial fading memory system, it is possible to partition the eigenvalues of matrix A in two sets, ξ_s and ξ_f related to slow and fast dynamics respectively, such that

$$\min_{\lambda \in \xi_s} |\lambda| > \max_{\lambda \in \xi_f} |\lambda| = \sigma_f. \tag{2}$$

Since all the eigenvalues in ξ_s are different from the ones in ξ_f, there exists a matrix T such that

$$A = T \begin{bmatrix} H_s & 0 \\ 0 & H_f \end{bmatrix} T^{-1} \tag{3}$$

where H_s, H_f are Jordan blocks that represent the slow and fast dynamics of the system respectively. That is, the eigenvalues of H_s and H_f are contained in ξ_s and ξ_f respectively. The slow and fast projections, denoted z_s and z_f, are obtained from

$$z = \begin{bmatrix} z_s \\ z_f \end{bmatrix} = T^{-1} x \tag{4}$$

where $z \in Z \subset \mathbb{R}^{n_x}$. The following property shows that if $\sigma_f < 1$, then system (1) satisfies Assumption 1. Moreover, the proof that we present for this property provides a procedure to obtain matrices C_s and C_f.

Property 1 (Fading parameter in linear systems). *Suppose that the eigenvalues of matrix A are partitioned in two set as in (2) with $\sigma_f < 1$, then system (1) is a partially fading memory system with fading parameter σ_f.*

Proof. Suppose that the same first control input $u(0)$ is applied to different initial conditions $x_a(0), x_b(0)$. We have,

$$\begin{aligned} x_a(1) - x_b(1) &= Ax_a(0) + Bu(0) - Ax_b(0) - Bu(0) \\ &= A(x_a(0) - x_b(0)). \end{aligned}$$

From (3) we obtain

$$\begin{aligned} x_a(1) - x_b(1) &= T \begin{bmatrix} H_s & 0 \\ 0 & H_f \end{bmatrix} T^{-1}(x_a(0) - x_b(0)) \\ T^{-1}(x_a(1) - x_b(1)) &= \begin{bmatrix} H_s & 0 \\ 0 & H_f \end{bmatrix} T^{-1}(x_a(0) - x_b(0)), \end{aligned}$$

and taking into account (4)

$$z_a(1) - z_b(1) = \begin{bmatrix} H_s & 0 \\ 0 & H_f \end{bmatrix} (z_a(0) - z_b(0)).$$

Proceeding in a recursive way, we obtain that if the same control sequence is applied to both projected initial conditions

$$z_a(t) - z_b(t) = \begin{bmatrix} H_s^t & 0 \\ 0 & H_f^t \end{bmatrix} (z_a(0) - z_b(0)). \tag{5}$$

There are two possible cases depending on whether the maximum singular value of matrix H_f, denoted as $\bar{\sigma}(H_f)$, is equal or greater than the fading parameter.

(a) $\bar{\sigma}(H_f) = \sigma_f$.

This is the most general case and occurs, for example, when the eigenvalues of H_f are different.