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Legendre-Gauss Pseudo-spectral Method on Special Orthogonal Group for Attitude Motion Planning

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Abstract: Legendre-Gauss pseudo-spectral method is extended to the special orthogonal group for attitude motion planning of an object in three-dimensional space. First, on the special orthogonal group, a complete left-invariant rigid-body dynamical model for the object in body-fixed frame is established, including kinematical model and dynamical model. Second, for kinematical model, equivalent Lie algebra equation corresponding to the configuration equation is derived based on the left-trivialized tangent of local coordinate map. Considering the Lie algebra space is isomorphism to Euclidean space, Legendre-Gauss pseudo-spectral method is directly used to discretize the dynamical model. Finally, an algorithm for transcribing the attitude motion planning problem is proposed based on LG-PS method on the special orthogonal group. The simulation results illustrate that the proposed method is effective for planning the trajectory of an object rotating in three-dimensional space.

Keywords: optimal control; special orthogonal group; Legendre-Gauss; pseudo-spectral method; motion planning

1. INTRODUCTION

Attitude motion planning is an important technology used by some rotating rigid body in three-dimensional space for performing some tasks, so that it has long been a research focus. In order to continuously fuel consumption, as well as to enhance the performance of the motion planning method itself, such as speed, reliability, motion planning bring forth the new through the old. But nothing less than to so few categories: artificial potential field method (Rimon and Koditschek (1994)), sampling-based method (Bertsekas (2007)), maneuvering automated method (Gavrilets et al. (2001)) and optimal control method (Rao et al. (2009)). These methods describe an object as a mass point or a single rigid body. However, when a rotating object is considered as a single rigid body in three-dimensional Euclidean space, the rotation configuration of the object constructs some special Lie group called the special orthogonal group SO(3) (Bullo and Murray (1995)). The planning methods developed on Lie group are called Lie group methods.

In Lie group methods, there is no doubt that optimal control theory is the most natural framework for solving the motion planning problem. However, except the linear quadratic issues such simple question, analytical solutions are seldom available or even possible. As a result, more often than not, one resorts to numerical techniques. Numerical methods for solving optimal control problems are divided into two major categories (Betts (1998)): indirect methods and direct methods. In an indirect method, the calculus of variations and the Pontryagin maximum principle are used to determine the first-order optimality conditions of the original optimal control problem, so it converts the optimal control problem (OCP) to a two point boundary-value problem, including indirect shooting method, indirect multiple-shooting method, etc. The primary advantage of the indirect methods is the high accuracy and assurances the solution satisfies the necessary optimality conditions. However, indirect methods have significant disadvantages. First, the necessary optimality conditions must be derived analytically, for most problems this derivation is difficult; Second, the radius of convergence is typically small. Third, a guess for the costates must be provided. In a direct method, the state and/or control of the OCP is discretized in some manner and the problem is transcribed into a nonlinear programming problem (NLP), then the NLP is solved by well known optimization techniques, including direct shooting method, direct collocation method. Although being not as high precision as indirect methods, direct methods have the advantages that the optimality conditions do not need to be derived; they have a large radius of convergence. In direct methods, we resort to pseudo-spectral (PS) method, which is widely used in fluid mechanics, quantum mechanics, linear and nonlinear waves, aerospace, and other fields by virtue of its high accuracy, spectral (or exponential) convergence rates, and requirement for less computer memory under the same precision condition, etc. Typical pseudo-spectral method can be used to transcribe a continuous OCP into a discrete nonlinear programming problem (NLP), and it has been shown that solving the NLP derived from the pseudo-spectral transcription of Legendre-Gauss (LG) form is exactly equivalent to solving the discretized form of the continuous first-order necessary conditions. LG form is used for transcribing the continues optimal control problem into a discrete nonlinear programming problem, it does not suffer from a defect in the optimality conditions at the boundary points due to that the endpoints are not collocation points (Benson (2004)). However, when applied to rigid body dynamics directly,

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general pseudo-spectral method can not preserve Lie group structure. For this purpose, a numerical method on the special orthogonal group called Legendre-Gauss geometric pseudo-spectral method (LG-GPS) is proposed, which provides satisfactory accuracy, computational efficiency and preserves the essential Lie group structure. How to apply LG-PS method to dynamics system on the special orthogonal group is still an open question, which is the main topic in this paper.

In this paper, we establish a rigid-body dynamical model of an object in body-fixed frame on the special orthogonal group. For the left-invariance of the configuration equation, we develop a numerical method, called Legendre-Gauss geometric pseudo-spectral (LG-GPS) method. With respect to kinematics, due to that applying the general pseudo-spectral method directly to the configuration equation of the rigid-body dynamics could not preserve the Lie group structure of the solution of the equation, we transform the configuration equation on the special orthogonal group into an equivalent equations in its Lie algebra space. With respect to dynamics, considering the Lie algebra space is isomorphism to $\mathbb{R}^3$, we use the classical Legendre-Gauss pseudo-spectral method (LG-PS) directly to compute the velocities at the same discrete points as that of configuration. Next, the proposed method is used for an object's attitude motion planning. Finally, it is illustrated that the proposed method is effective for the motion planning through some typical simulation.

The rest of the paper is organized as follows. In section 2, a left-invariant rigid-body dynamical model of an object on the special orthogonal group is established. LG-GPS method on the special orthogonal group is proposed in section 3. Then, how to apply the proposed method to an object's off-line attitude motion planning problem is illustrated in section 4. Finally, an simulation example is provided in section 5 and concluded in last section.

2. DYNAMICAL MODEL OF AN OBJECT ON SPECIAL ORTHOGONAL GROUP

2.1 Left-invariant mechanical system in a body-fixed frame

The motion of a rigid body on Lie group G could be described in a space frame or a body-fixed frame. To avoid confusion, we would consider the motion of a single rigid body in a body-fixed frame in the sequel.

Definition 1 (left-invariant mechanical systems on Lie group G (Bullo and Murray (1995))). In a body-fixed frame, the motion of a rigid body could be described as a left-invariant mechanical system as follows:

$$\dot{g} = T_e L_g \circ \hat{\xi} \in T_g G, \quad \forall g \in G \tag{1}$$

where g denotes the configuration of a rigid body in G and $\hat{\xi} = T_{g^{-1}} \dot{g} \in T_e G$ denotes the velocity of the rigid body in the body-fixed frame, where $T_e G := \mathfrak{g}$, known as the Lie algebra, is the tangent space at the identity e of G and "$\wedge$" is the hat operator satisfying isomorphism $\wedge : \mathbb{R}^n \to \mathfrak{g}$ with "$\vee$" being its inverse mapping. $L_g : G \to G$ represents the left translation; that is, $L_g(h) = gh, \quad \forall h \in G$, and $T_e L_g$ is its tangent space at e. For the special orthogonal group, (1) is equivalent to $\dot{g} = g \cdot \hat{\xi}$. "Left-invariant" implies that the mechanical system is invariant under left multiplication by constant matrices.

2.2 Kinematical model

The left-invariant kinematics of attitude motion of a object on the special orthogonal group are given by (Hussein et al. (2006); Kobilarov and Marsden (2011))

$$\dot{R} = R\hat{\omega} := G(\omega, R) \tag{2}$$

where R and ω denote the attitude configuration and angular velocity of a rotating object on the special orthogonal group. The hat operator $\wedge : \mathbb{R}^3 \to \mathrm{so}(3)$, for all $y \in \mathbb{R}^3$, satisfying $\hat{\omega} \cdot y = \omega \times y$, where $\times$ is vector cross product.

2.3 Dynamical model

It is known that the attitude dynamics of the object are generally satisfying (Hussein et al. (2006); Kobilarov and Marsden (2011))

$$\dot{\omega} = J^{-1}\left[J\omega \times \omega + R^* \partial_R V(R) + T_{ext} \right] := F(\omega, R, T_{ext}) \tag{3}$$

where J is referred to as the inertia matrix of the rigid body, $R^* \partial_R V(R)$ denotes the moment of the conservative force with $V(R)$ being potential energy, and $T_{ext} := B \cdot u$ is that of the nonconservative external force with being the control conjugate vector.

3. LG-PS ON SPECIAL ORTHOGONAL GROUP

3.1 Configurations at the LG points and the endpoint

In order to transform (2) on the special orthogonal group to equivalent differential equation evolving on its Lie algebra, we briefly describe the basic idea of equivariant map.

Proposition 2 (Equivalent configuration on $\mathrm{so}(3)$ of left-invariant configuration). The configuration (2) has the following equivalent equation on $\mathrm{so}(3)$.

$$\dot{u} = \mathrm{d}f_{-u}^{-1}(\omega) := \tilde{G}(\omega, u) \tag{4}$$

where $f : \mathrm{so}(3) \to G$ is a local coordinate map on the special orthogonal group, $\mathrm{d}f$ the differential of the coordinate map.

Proof. We assume that the solution R of (2) can be written as the following form,