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Interval sliding mode observer design for linear and nonlinear systems

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ABSTRACT

In this paper, an interval sliding mode observer design method for uncertain systems is proposed. Uncertainty is assumed between a known minimum value and a maximum value. The observer is then constructed via a convex weighted sum of an upper estimator corresponding to the maximum value of the uncertainty and a lower estimator corresponding to the minimum value of the uncertainty. The weighting factor is calculated at each time, from the different measured outputs and the bounds of the interval of the estimator.

1. Introduction

The design of an effective control strategy often requires knowledge of the state variables of the dynamic system. However, the state variables rarely not accessible to measurement. The state observers developed several decades ago by Luenberger and Kalman are enable to reconstruct the state of the model if the parameters supplied are well known for linear systems [1]. On the other hand, for nonlinear systems, there exist several kinds of observers to be used depending on the mathematical structure of the process model and the available information. For example, we have the extended Kalman filter [2], the high gain observer [3] and the sliding mode observer [4]. All these approaches are more or less robust with respect to the disturbances and the measurement noise. However, they often provide unsatisfactory estimations in the presence of uncertainties in model parameters [5]. These uncertainties can be serious limitations in the application of these observers which provide biased estimates. In most cases, there is partial information on uncertainties, including that giving the maximum and minimum limits. It would be interesting to take advantage of this partial information to implement a robust observer [6].

To overcome this kind of problems, several methods have been developed recently in the set membership framework. Their principle consists in computing sets guaranteed to contain all the state vectors even in presence of uncertainties. These approaches are based on the analysis of uncertain systems, themselves based on special geometrical forms, such as ellipsoids [7], zonotopes [8], and interval vectors [9,10].

The technique based on the interval arithmetic introduced by Moore [11] is an interesting alternative for the design of observers for the systems subject to partially unknown uncertainties. Currently, there is a large number of works devoted to the problem of state estimation and parameters using interval analysis [12–14].

Interval observers are highly relevant in the context of observation of systems for which only a poor model is available, like in fields of ecology, epidemiology or biology; see, e.g. [15]. Interval observers have been already considered for systems with unknown inputs [16], linear systems [17,18], uncertain systems [19–21], time-varying systems [22], delay systems [23,24], nonlinear systems [25], linear parameter varying systems [26], discrete-time systems [27], and for stabilization [28].

Besides, the sliding mode techniques have become very popular for the design of observers for linear and nonlinear systems [29–32]. The sliding modes ensure a finite time of the estimation error convergence to zero and complete insensitivity to a matched uncertainty [33–35]. The objective of this work is to combine both approaches, i.e. the interval observers and the sliding mode techniques, in order to improve the accuracy of estimation achieved by interval observers. This combination leads to a significant decrease of the interval estimation conservatism.

In spite of the results already available in the literature, much remains to be done to complete the theory of interval observers. In [6], the authors propose a weighted average observer obtained by a weighted convex sum of the upper and lower estimators. This

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estimator is based on the Luenberger observer. The Luenberger estimator is calculated as a weighted value between the min and the max observers. The weighting factor is calculated from measured output and the bound of the interval of the estimator.

Nowadays and to our knowledge, few studies have been made to design interval sliding mode observers; see, e.g. [36,37]. Our contribution is to design an interval sliding mode observer for linear and nonlinear dynamic systems. This observer is calculated as a weighted average of the upper and the lower sliding mode estimator. The interval observer consists in two estimators, an upper estimator and a lower estimator. The drawback of the interval observer is that it provides an estimated interval rather than a unique estimated value.

Various applications of interval analysis in the literature include the treatment of uncertainty in the optimal design of chemical and biological plants. Proper control of biological and chemical systems strongly depends on reliable input information, which is usually obtained from fast and simple measurements or estimated from mathematical structures called observers. Uncertainty is a central concept when dealing with biological and chemical systems because they are inherently subject to large natural variations. Uncertainty is recognized as an important part of the analysis of control strategies for biological and chemical systems [38–40]. Input data uncertainties have also been recognized as a key problem in accurate modeling [41]. In the literature, a large number of works on the computation of interval observers for uncertain systems takes into account the uncertainties on control inputs, as well as on the dynamics of the system [42–44]. With increasing complexity of models, it also becomes important to analyze the model parameter sensitivity while taking into account uncertainties in the input and calibration data. In this work, we develop a framework for the quantification of the impact of uncertainties in the model inputs.

This paper is organized as follows. Section 2 presents the weighted average of the interval sliding mode observer for a class of linear systems. Section 3 presents a generalization to nonlinear systems. To illustrate the derived results, numerical examples are given in Section 4. Finally, Section 5 gives conclusions and some perspectives of the present work.

2. Notations and definitions

In this paper, the following notations and definitions will be used.

- $\mu_{n\times m}$ is the set of real matrices with n lines and m columns.
- $I_n \in \mu_{n\times n}$ denotes the identity matrix.
- $\mathbb{R}$ denotes the set of real number.
- $\mathbb{R}^+$ denotes the positive set of real number.
- $\|x\| = \sqrt{\sum x_i{}^2}$ is the Euclidian norm of $x \in \mathbb{R}^n$.

Definition 1. A real matrix $M \in \mu_{n\times m}$ is called a Metzler matrix (or cooperative) if all its off-diagonal entries are nonnegative, i.e. $M_{ij} \geq 0, i \neq j$. M_{ij} denotes the (i, j) element.

We now give the definition of cooperative dynamical systems. Consider the following nonlinear dynamic system defined in the open subset $\mathcal{D}$ of $\mathbb{R}^n$

$$\begin{cases} \dot{x}(t) = f(x(t), t) \\ x(t_0) = x_0 \end{cases} \tag{1}$$

where $x(t) \in \mathcal{D} \subset \mathbb{R}^n$ is the n-dimensional state vector, $t \in \mathbb{R}_+$ denotes the time and t_0 the initial time. The vector field $f : \mathcal{D} \times \mathbb{R}_+ \to \mathbb{R}^n$ is assumed to be sufficiently differentiable with respect to $x \in \mathcal{D}$.

Definition 2. System (1) is said to be cooperative in $\mathcal{D} \subset \mathbb{R}^n$ if the differentiable vector field $f : \mathcal{D} \times \mathbb{R}_+ \to \mathbb{R}^n$ is such that the Jacobian matrix $\frac{\partial f(x,t)}{\partial x}$ is Metzler for all $x \in \mathcal{D}$ and for all $t \geq t_0$. Consequently the following linear time-invariant system

$$\dot{x}(t) = Ax(t)$$

is cooperative if the state matrix A is Metzler.

In the rest of the paper we consider $\mathcal{D} \equiv \mathbb{R}^n$.

Lemma 1. [45,46]

$sign(a+b) \leq sign(a) + sign(b) + 1$ for all a and b. with $a \in \mathbb{R}$ and $b \in \mathbb{R}$.

Property 1. *The comparison principle ([47,48])*

Let two cooperative systems be

$$\begin{cases} \dot{x}(t) = f(x(t), t), \;\; x \in \mathbb{R}^n \\ \dot{\tilde{x}}(t) = \tilde{f}(x(t), t), \;\; \tilde{x} \in \mathbb{R}^n \end{cases} \tag{2}$$

with the initials conditions $x_i(t_0) = x_{0i}$ and $\tilde{x}_i(t_0) = \tilde{x_{0i}}$, $i = 1, 2, \ldots, n$ respectively, which satisfy $(x_{0i} \leq \tilde{x}_{0i})$ and $(f_i(x(t), t) \leq (\tilde{f}_i(x(t), t))$, $i = 1, 2, \ldots, n$, then $x_i(t) \leq \tilde{x}_i(t)$, $i = 1, 2, \ldots, n$, $\forall t \geq t_0$.

3. Sliding mode interval observer for linear systems

Consider a linear time invariant system described by the system of Eqs. (3)

$$\begin{cases} \dot{x}(t) = A\;x(t) + \theta\;B\;u(t) + D\;v(t) \\ y(t) = C\;x(t) \end{cases} \tag{3}$$

with: $x(t_0) = x_0$.

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}$, $y \in \mathbb{R}$ are the state, the input and output. $v \in \mathbb{R}^q$ represents the unknown disturbance and θ represents the uncertainty parameter for which the lower and upper bounds are known, that is $\theta \in \left[\theta_w, \theta_z\right]$.

θ_w and θ_z are the known lower bound and upper bound respectively.

The matrices $A \in \mu_{n\times n}$, $B \in \mu_{n\times 1}$, $D \in \mu_{n\times q}$ and $C \in \mu_{1\times n}$ are constant matrices. We take the following assumptions.

Assumption 1. (A, C) is observable.

Assumption 2. The matrix A is cooperative.

Assumption 3. The input $u(t)$ and the unknown bounded disturbance $v(t)$ are positives for all $t \in \mathbb{R}^+$.

The lower and the upper bound of $v(t)$ are known, that is

$$\forall t > t_0 : 0 < u_{\min} < u(t) \tag{4}$$

$$\forall t > t_0 : 0 < v_{\min} < v(t) < v_{\max} \tag{5}$$

Assumption 4. θ is an unknown parameter which satisfies the following conditions

$$\theta_w\;B\;u(t) < \theta\;B\;u(t) < \theta_z\;B\;u(t) \tag{6}$$

Theorem 1. *Let Assumptions 1–4 be satisfied. If there exists a pair of cooperative systems*

$$\dot{w}(t) = A\;w(t) + \theta_w B\;u(t) + L(y - C\;w) + K_s(\text{sign}(y - C\;w)) + Dv_{\min} \tag{7a}$$

$$\dot{z}(t) = A\;z(t) + \theta_z B\;u(t) + L(y - C\;z) + K_s(\text{sign}(y - C\;z)) + Dv_{\max} \tag{7b}$$

with given initial conditions $w(t_0)$, $z(t_0)$ and satisfying the following conditions