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An extremum-seeking control method driven by input–output correlation

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ABSTRACT

This paper presents a new extremum-seeking controller (ESC) derived from a statistical interpretation of conventional ESC. The proposed ESC replaces the gradient feedback variable with a normalized correlation coefficient. The algorithm, as presented, can be fully configured with knowledge of only the open-loop response time of the system being optimized and the amplitude of the dither signal. Simulation tests show that the new algorithm has fast convergence times compared with other types of ESC algorithms. The paper also shows that the proposed method is not limited to a periodic dither signal and that it can also utilize a stochastic signal to similar efficacy. The purpose of the paper is to describe the new algorithm and present a practical implementation along with results from simulation tests.

1. Introduction

Extremum-seeking control (ESC) is an adaptive control method that aims to optimize a measure of performance in a system by adjusting one or more system inputs/parameters in an automated fashion. For the case of one system input, conventional ESC is used to find the level of the input that corresponds to the maximum (or minimum) of the performance measure. This is achieved by applying a persistent disturbance (often called *dither*) to the system input and calculating a quantity proportional to the derivative of the performance with respect to the manipulated input. The purpose of the ESC method is to change the level of the input to a point such that this derivative is zero. A succinct review and history of ESC is provided in [25]. ESC is extendable to problems where multiple variables affect the performance variable. In this case, the goal becomes to drive the partial derivatives of the performance variable with respect to each input to zero.

Research focused on ESC has gained in popularity since Krstic and Wang provided the first rigorous stability proof for a general nonlinear (non-affine in control) system [8]. There have been several subsequent theoretical advances (e.g., [26,20,18,14,17,5]) and also a large increase in the number of applications since this paper was published such as those described in [27,6]. From the perspective of a practitioner, ESC is appealing because it requires less system knowledge than alternative model-based approaches. However, for low-cost applications such as the systems found in buildings, including, but not limited to HVAC (heating, ventilating, and air-conditioning), the configuration requirements can still be a barrier to adoption. For these applications, even minor configuration costs can outweigh the benefits as discussed in [2,22,21].

A conventional ESC strategy is depicted in Fig. 1. The plant output (performance measure) is passed through a high-pass filter $H_{hp}(s)$ to remove the DC component of the signal and the output of the high-pass filter is multiplied by (a phase-shifted) dither signal, which is usually sinusoidal. The result of the multiplication is proportional to the derivative of the performance measure with respect to the manipulated variable and a loop that includes an integrator and low-pass filter is closed around this value. The plant input is calculated by adding a perturbation signal to a value that is changed by the feedback loop with the objective of driving the derivative to zero.

The conventional ESC strategy has a number of elements that need to be tuned for each different system and improved performance can be achieved by incorporating greater prior knowledge of the plant being optimized e.g., [24]. Most of the elements need to be configured based on the dynamic characteristics of the plant. In particular, the high- and low-pass filters as well as the frequency of the perturbation signal need to be configured according to the

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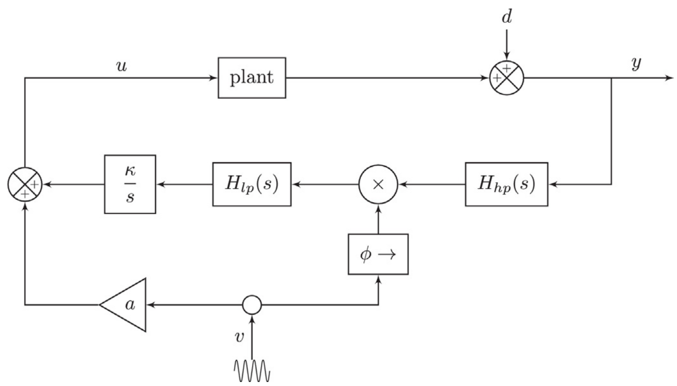

Fig. 1. Block diagram of a conventional ESC algorithm.

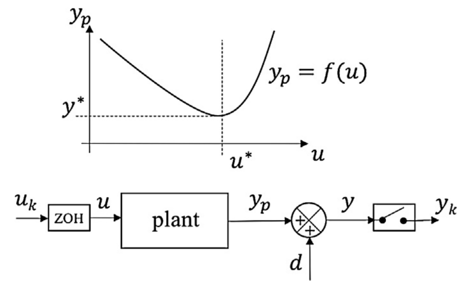

Fig. 2. SISO plant.

plant dynamics. In general, the ESC algorithm has to be configured so that the gradient-descent occurs at a slower rate than the open loop dynamics of the plant thus ensuring time-scale separation.

Two ESC parameters need to be set according to expectations about the range of the system performance measure: the amplitude of the dither signal and the gain of the integrator κ. The amplitude of the dither signal will be constrained by the physical range of the manipulated variable and also by the size of the disturbance that is acceptable to plant operators. However, this amplitude will also need to be set so that it is large enough to overcome any measurement noise d contained in the performance variable y so that the signal to noise ratio is sufficiently high. Operational constraints will nevertheless define the upper bound on the dither amplitude in most practical systems. Determination of the integrator gain, however, cannot be guided by operational constraints and will instead require knowledge of the gain and dynamics of the plant [5].

The main contribution of this paper is to derive an implementation of ESC that requires minimal configuration. In order to achieve this, conventional single-input ESC strategy is interpreted in a statistical framework. This interpretation leads to the development of an algorithm based on sample statistics, which are calculated using exponentially-weighted moving averages (EWMAs). The tuning problem is addressed by deriving a statistical feedback variable that is naturally scale-independent. This removes the need to obtain information about the static gain of the plant and reduces the tuning to only requiring knowledge of an open-loop response time of the plant. We will refer to this open-loop response time as the *plant time constant*. This kind of first-order characterization of the plant dynamics is popular in practice and an estimate for a particular plant could be obtained either from expert knowledge or by carrying out an open-loop step test.

The algorithm presented in this paper is designed to be indifferent to the type of perturbation and is thus not restricted to a periodic dither. We have found that operators do not like to see periodic disturbances enter the plant, mainly because they are perceptible and also because they can be confused with poorly tuned controllers. We demonstrate that a stochastic perturbation can be used with the proposed method to a similar efficacy as a periodic dither. The use of stochastic disturbances in ESC strategies has been studied by several authors and has been shown to be an effective type of perturbation, e.g., [12,14,10].

The paper is structured as follows: Section 2 describes the proposed ESC algorithm derived from a statistical interpretation of conventional ESC. Section 2.4 discusses different types of possible perturbation signals and presents guidelines for the use of sinusoidal and stochastic signals. Section 3 presents results of tests on a simplified simulation model and also on a more detailed simulation model of an air-conditioning system. Conclusions are drawn in the final section.

2. Description of the method

2.1. Preliminaries and assumptions

This paper focuses on a single-input–single-output (SISO) plant as depicted in Fig. 2. Extensions to multivariable problems are not considered here and are left to future work. For the SISO case, the plant has an input u with output y_p. The plant output is subject to a disturbance signal d and the measurable signal is y. The plant has a static map $y_p = f(u, p)$ that depends on u and also other possible effects that are consolidated in p. It is assumed that the static map exhibits an extremum point depicted in the figure as $y^*(p) = y(u^*(p), p)$. The purpose of ESC, in general, is to manipulate u in order to locate $u^*(p)$. The advantage of ESC is that it does not require explicit knowledge of the plant, the static map $f(\cdot)$ or p. The method presented in this paper is in a discrete-time form and thus interfaces to the plant via a zero-order hold on the input and a sampler on the output. Without loss of generality, maximization or minimization is handled by changing the sign of y.

To set context for the application, the plant dynamics can be described in the following general non-linear state-space form:

$$\dot{x} = g(x, u), \qquad y_p = h(x), \qquad y = y_p + d(t) \tag{1}$$

where x is a state variable of n dimensions.

Assumption 1. The static map $f(u)$ is assumed to be convex with an extremum point at (u^*, y_p^*).

Assumption 2. For each $u \in \mathbb{R}$ the equilibrium x is asymptotically stable.

Assumption 3. For a set of samples $\mathbf{u} = \{u_{k-j}, \ldots, u_{k-1}, u_k\}$, the variance of $\mathbf{u}$ is finite and positive, thus: $\mathrm{Var}(\mathbf{u}) \in \mathbb{R}_{>0}$; we further assume that $\mathrm{Var}(\mathbf{y}) \in \mathbb{R}_{>0}$. The condition of stationarity is not demanded as a property of the signals because the sample statistics will approximate average properties over the considered sample set. However, all subsequent analysis assumes the application of a stationary dither signal.

2.2. Statistical interpretation of conventional ESC

If we consider a discrete-time implementation of the conventional ESC depicted in Fig. 1 and focus on a point in time k, we can recognize the following:

- assuming that the high-pass filter is designed to remove the DC value of the signal, the output of the high-pass filter is: $y_k - \bar{y}$, where $\bar{y}$ is the sample mean of y.
- the result of the multiplier is: $m_k = (y_k - \bar{y})(v_k)$, where $\mathrm{E}[v] = 0$.