



Iterative learning Kalman filter for repetitive processes



Zhixing Cao^{a,b}, Jingyi Lu^a, Ridong Zhang^a, Furong Gao^{a,c,d,*}

^a Department of Chemical and Biomolecular Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong Special Administration Region

^b Harvard John A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138, United States

^c Fok Ying Tung Research Institute, Hong Kong University of Science and Technology, Guangzhou, China

^d Kunda Mold (Shen Zhen) Co. Ltd., Shenzhen, Guangdong 518129, China

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ABSTRACT

In this paper, a discrete-time iterative learning Kalman filter scheme is proposed for repetitive processes to reject repeatable disturbances as well as random noises. The proposed state estimator scheme integrates Kalman filter with iterative learning control. The estimation process contains two stages: a conventional Kalman filter is applied in the first stage; the second stage refines the estimates in an iterative learning fashion, leading to a gradual improvement on the estimation performance. According to the estimates that the first stage feeds to the second stage, the optimal design includes two types – posterior type and priori type. In order to reduce the memory and computation load of the optimal design, two suboptimal estimators are provided as well. The stability of the both suboptimal estimators is also studied. Furthermore, a lower bound is given to estimate the ultimate estimation performance before implementing any estimation. Finally, an illustrative example of injection molding is given to verify the performance of the four estimators developed.

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1. Introduction

State estimation has always been a fundamental problem in control theory, and has many applications in various areas, e.g., electric power systems [1,2], fault diagnosis [3–5]. State estimation has, therefore, attracted lots of interests in control academia and witnessed numerous achievements [6–8]. Among these achievements, Kalman filter [9], a recursive state estimator, is the most ubiquitous approach, which has found wide applications in many industrial or military areas, e.g., unmanned vehicle [10], GPS positioning [11], and image recovery [12]. An underlying assumption of no matter Kalman filter or its variants, i.e., extended Kalman filter (EKF) [8] and unscented Kalman filter (UKF) [8,13], is the entirely random noise, especially white noise. It can be, however, intuitively expected that the estimation performance may be conservative and can be further improved, if the noise is not random wholly but with certain “structures”. One of the popular “structures” is the repetitiveness. In reality, repetitive processes are an important class of industrial processes and have wide applications, i.e., batch process (for example, injection molding [14] and semi-conductor process [15]), continuous process with periodic input and periodic continuous process (for instance, rotary system [16] and long-wall coal cutting [17]). Iterative learning control (ILC), categorized as an intelligent control methodology, is good at utilizing such repetitiveness to reject disturbances, thus improving transient period control performance within finite intervals, even without precise knowledge of plant [18]. For more details on it, please refer to [17–24].

In this paper, we propose *iterative learning Kalman filter (ILKF)* to improve state estimator's performance associated with repetitive processes, by integrating Kalman filter and ILC. *Specifically, our work contains three main contributions as stated below.*

First, we introduce iterative learning mechanism into a state estimator for repetitive processes. Basically, the estimation process consists of two stages. In the first stage, a conventional Kalman filter is employed to estimate the current batch's¹ deviation from the last batch's state trajectory. It generates a “coarse guess” by adjusting the previous batch's state estimate by the deviation estimate. The second stage “polishes” the “coarse guess” by invoking the iterative learning mechanism. Relevant results concerning ILC in the stochastic setup are

* Corresponding author at: Department of Chemical and Biomolecular Engineering, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong Special Administration Region.

E-mail address: kefgao@ust.hk (F. Gao).

¹ A sequence of repeated actions in a fixed time is called a batch, cycle, trial or shot [18].

[25,26]. In [25,26], the author studied the problem of properly selecting a learning gain of the control algorithm, but did not consider the real-time feedback information, which differs from our method. From the point of view of observer design, Alvarado and his coworkers proposed an observer for repetitive nonlinear systems by approximately minimizing the gap between measurement and estimation in a moving-horizon fashion [27]. Hatonen and Moore proposed an iterative learning observer for repetitive processes in a lifted system frame by stacking all the states in a batch into a super-vector [28]. Both papers above only studied the deterministic system rather than in a stochastic setup as in this paper.

Second, more than presenting two types of optimal estimators, two suboptimal state estimators are developed as well, intending to alleviate the memory and computation demand involved in the optimal estimator. The stability of both suboptimal estimators has been analyzed as well.

Third, a lower bound of error covariance has been provided for roughly estimating the ultimate performance before implementing any estimation experiment.

A related work has been done in Cao et. al. [29], where only the priori type has been analyzed. This current paper not only studies the priori-type ILKF, but also its posterior type, which is more difficult to analyze, and this paper's result outperforms the priori type. In addition, this work also proposes two suboptimal estimators to reduce the memory and computation load. Besides, it should be noted that we employ an approach to reject the repeatable disturbance instead of introducing a disturbance estimator. The reasons are: first, disturbance estimator usually requires an additional model as to disturbance, which might not be available in some circumstances; second, even if disturbances can be estimated, its estimation will be compensated by feedforward, while our approach takes a feedback way.

The remainder of the paper is organized as follows: Section 2 formulates the problem; Section 3 presents the optimal estimator design including an *posterior* type and an *priori* type; Section 4 develops and analyzes two suboptimal estimators; an illustrative example of injection molding is given in Section 5; conclusions and outlooks are presented in Section 6.

The following notations will be used throughout the paper: superscript T stands for transpose; the symbol $\triangleq$ signifies “defined as”; $\mathbb{E}[\bullet]$ means the expectation of $[\bullet]$; δ is the batch-wise backward difference operator; $\mathbb{I}[a, b)$ represents all the integers greater or equal to a and less than b ; I is the identity matrix with compatible dimensions; $\text{Co}\{\bullet\}$ signifies a linear combination of $\{\bullet\}$; $\rho_{\max}(\bullet)$ is the spectral radius of $(\bullet)$; $\text{rank}(\bullet)$ represents the rank of $(\bullet)$; $\rho_{\min}(\bullet)$ is the modulus of the minimum eigenvalue of $(\bullet)$; $\text{tr}(\bullet)$ is the trace of $(\bullet)$; $\text{avg}(\bullet)$ is the average of $(\bullet)$; A^{T+} means $A + A^T$.

2. Problem formulation

Within the paper, we consider the following linear time-varying system:

$$\begin{cases} x_k(t+1) = A_t x_k(t) + d(t) + w_k(t) \\ y_k(t) = C_t x_k(t) + v_k(t) \end{cases} \quad (1)$$

where $x_k(t) \in \mathbb{R}^n$ and $y_k(t) \in \mathbb{R}^l$, $l \leq n$ are respectively state vector and output vector of the system at the k -th batch, $k \in \mathbb{I}[0, +\infty)$ and the t -th time, $t \in \mathbb{I}[0, L]$. L is the duration of each batch. $A_t \in \mathbb{R}^{n \times n}$, and $C_t \in \mathbb{R}^{l \times n}$ is full row rank. Regarding the system in (1), we impose the following assumptions.

- (A1) $w_k(t)$ and $v_k(t)$ are process noise and measurement noise respectively, and assumed to be independent two dimensional white noise, i.e.,

$$\begin{aligned} \mathbb{E}[w_k(t)] &= \mathbb{E}[v_k(t)] = 0 & \mathbb{E}[w_i(j)v_k^T(t)] &= 0 \\ \mathbb{E}[w_j(i)w_k^T(t)] &= Q\delta_{i,t;j,k} & \mathbb{E}[v_j(i)v_k^T(t)] &= R\delta_{i,t;j,k} \end{aligned}$$

for any k, t, i, j , where $\delta_{i,t;j,k}$ is two dimensional Kronecker delta $\delta_{i,t;j,k} = 1$ if and only if $i = t$ and $j = k$, otherwise $\delta_{i,t;j,k} = 0$. Additionally, Q and R are positive definite matrices. Thus, according to Cholesky decomposition, we have $Q = MM^T$ with M being a lower triangular matrix.

- (A2) For $t \in \mathbb{I}[0, L]$, the pair (A_t, C_{t+1}) is uniformly observable; the pair (A_t, M) is uniformly controllable [30].
- (A3) For any t , $\rho_{\max}(A_t) < 1$ holds.
- (A4) The initial condition of each batch $x_k(0)$ is subject to identical independent normal distribution $\mathcal{N}(0, \Gamma)$.
- (A5) $d(t)$ represents the unknown repeatable disturbance and is bounded. Mathematically, it is a only a function of t and k -invariant and $\sup_{t \in \mathbb{I}[0, L]} \|d(t)\| < \infty$.

Remark 2.1. Note that the system (1) together with A1, A2, A4 is a standard setup for state estimation problem, e.g. [30,31]. A5 only assumes that $d(t)$ is a repeatable and bounded disturbance. Practically, we can approximate $d(t)$ as stochastic noise, i.e. $d(t)$ approximately subject to normal distribution $\mathcal{N}(\text{avg}[d(t)], \Gamma_d)$, $\Gamma_d \triangleq \max_{t \in \mathbb{I}[0, L]} \{d(t) - \text{avg}[d(t)]\} \{d(t) - \text{avg}[d(t)]\}^T$. A4 is a common assumption adopted in many control and estimation literatures; and it assures the boundedness of the state variance. Let $\rho \triangleq \max_{t \in \mathbb{I}[0, L]} \rho_{\max}(A_t)$; and then it is easy to have $\sum_{j=0}^{t-1} \rho^{2j}(\Gamma_d + Q) + \rho^{2t}\Gamma$ is an upper bound of $\mathbb{E}[x_k(t)x_k^T(t)]$ for any k . It follows that

$$\bar{P} \triangleq \max_{t \in \mathbb{I}[0, L]} \sum_{j=0}^{t-1} \rho^{2j}(\Gamma_d + Q) + \rho^{2t}\Gamma$$

is bounded, since $\rho < 1$. Obviously, it is an upper bound of $\max_{t \in \mathbb{I}[0, L]} \mathbb{E}[x_k(t)x_k^T(t)]$. Recall that the majority of ILC literatures impose the *identical*

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